

CS103
WINTER 2026



Lecture 14:
Finite Automata

Part 1 of 3

Finite Automata

Part 1

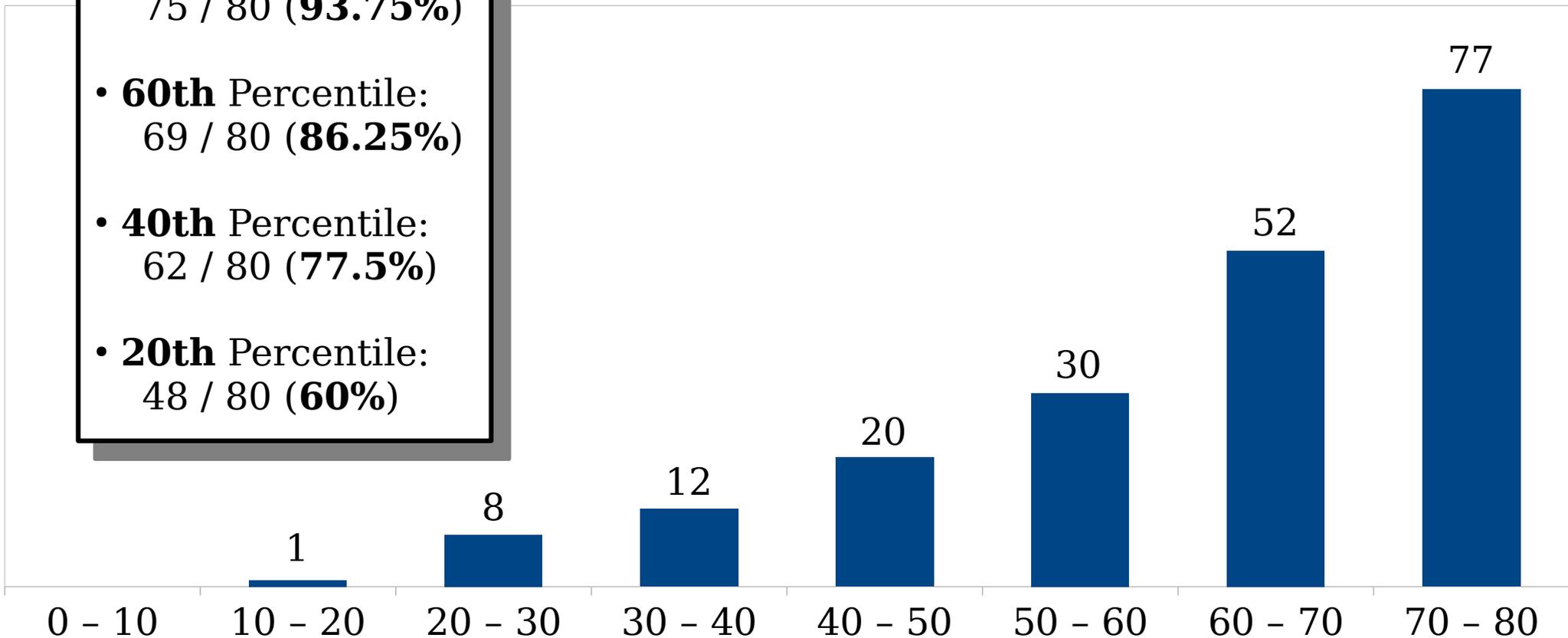
1. Midterm Results
2. Computability Theory: The Central Question
3. Toward a Model of Computation
4. Automata
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7. DFAs
8. Designing DFAs
9. What's Next?

Finite Automata

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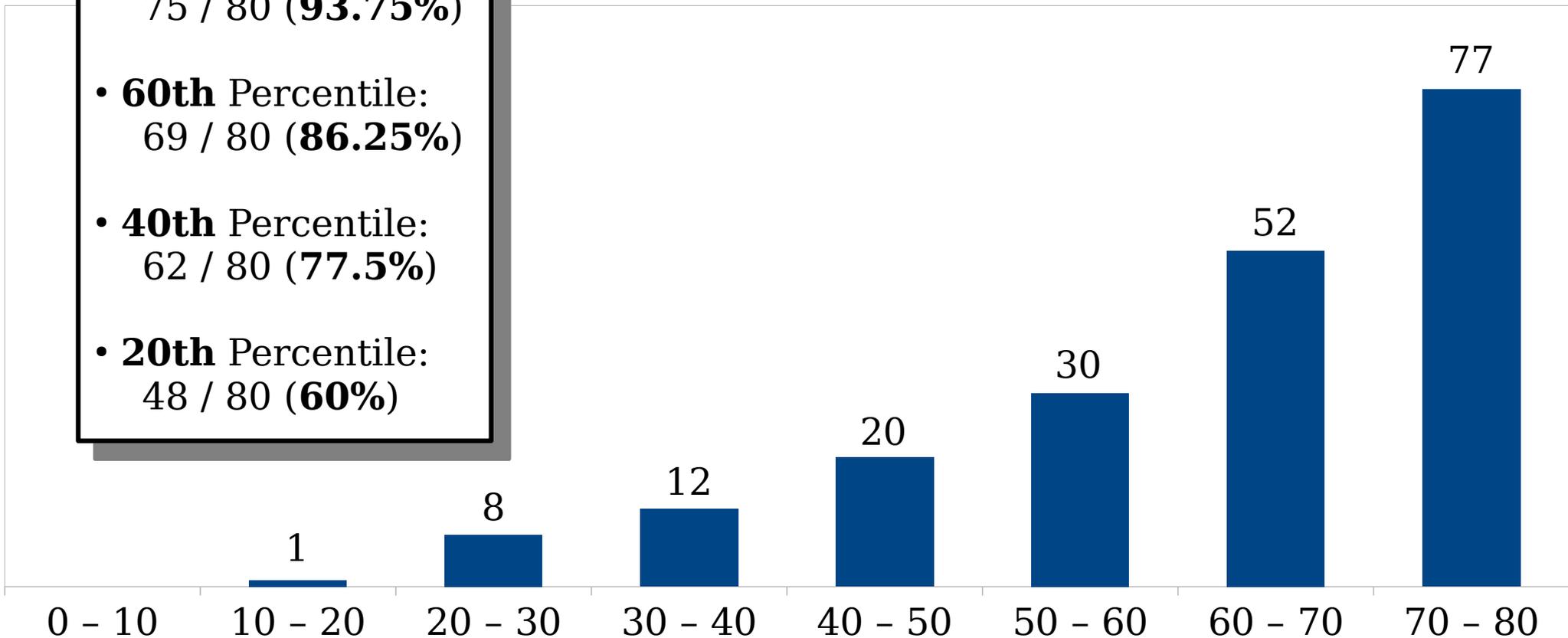
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- **80th** Percentile:
75 / 80 (**93.75%**)
- **60th** Percentile:
69 / 80 (**86.25%**)
- **40th** Percentile:
62 / 80 (**77.5%**)
- **20th** Percentile:
48 / 80 (**60%**)



- If your score isn't in the range you're used to, ask yourself what happened, and be thoughtful and honest with yourself.
- Approach conversations with course staff genuinely. We approach them from a place of non-judgment.
- If anything about the grading feels arbitrary, that might be a symptom that you're overlooking a key concept. Come talk with us so we can help clarify!

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48 / 80 (**60%**)



- We want everyone to be wildly successful!
 - Review feedback
 - Assess (small scattered point losses? one large loss?)
 - 1-on-1s (see Anisha's Ed post)
- Assuming comfort going forward:
 - Contrapositive
 - Negations (implication, quant.)
 - Assume/Prove table
 - Proofwriting Checklist

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Computability Theory

What problems can we solve with a computer?

Computability Theory

What problems can we solve with a computer?

What kind of
computer?



Two Challenges

- Writing proofs on formal definitions is hard, and computers are *way* more complicated than sets, graphs, or functions.
- Computers are dramatically better now than they've ever been, and that trend continues.
- **Key Question:** How can we prove what computers can and can't do...
 - ... without multi-hundred page proofs?
 - ... so that our results are still true in 20 years?

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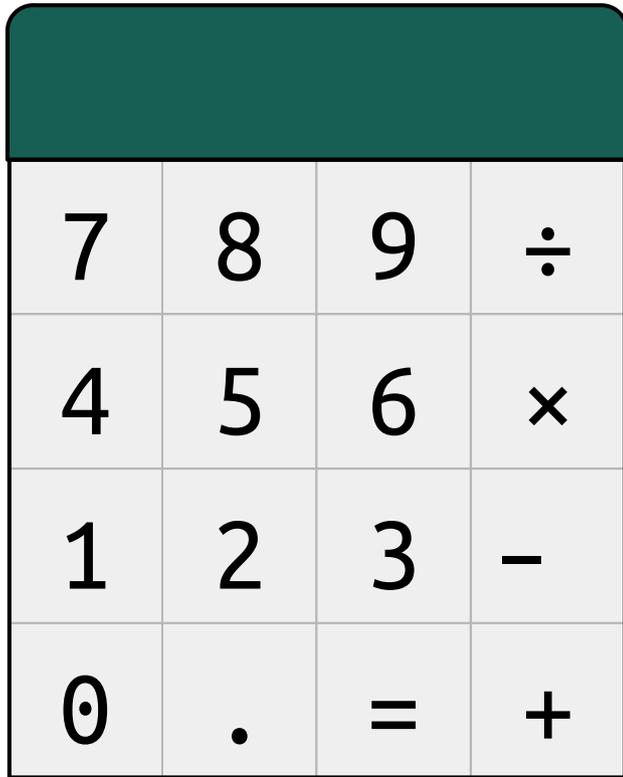
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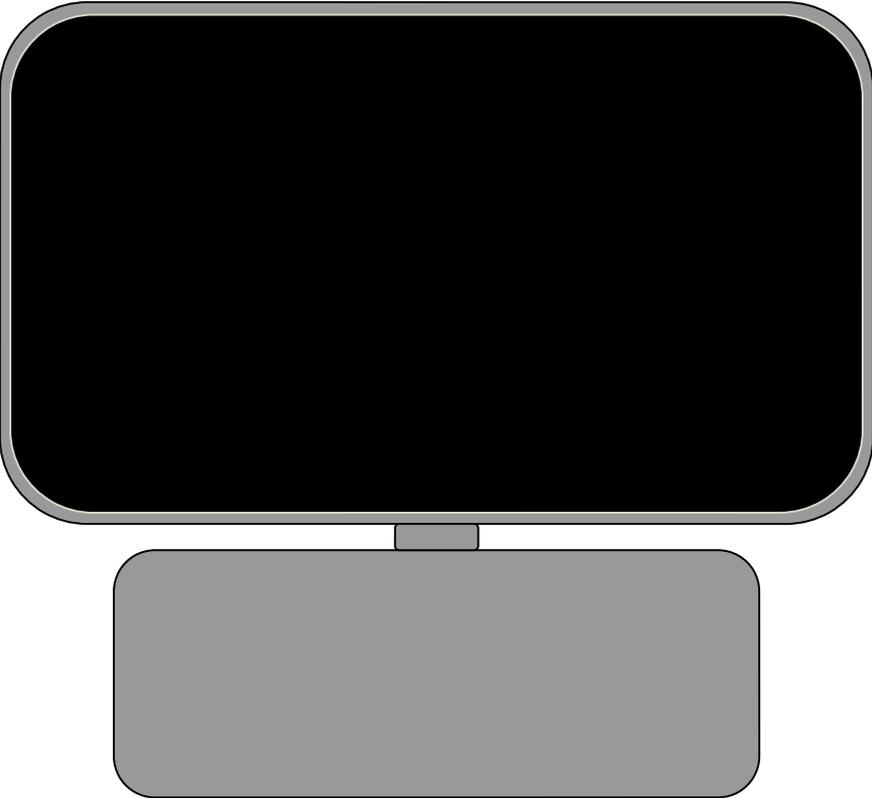
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Why does this computer "feel" less powerful...



...than this one?

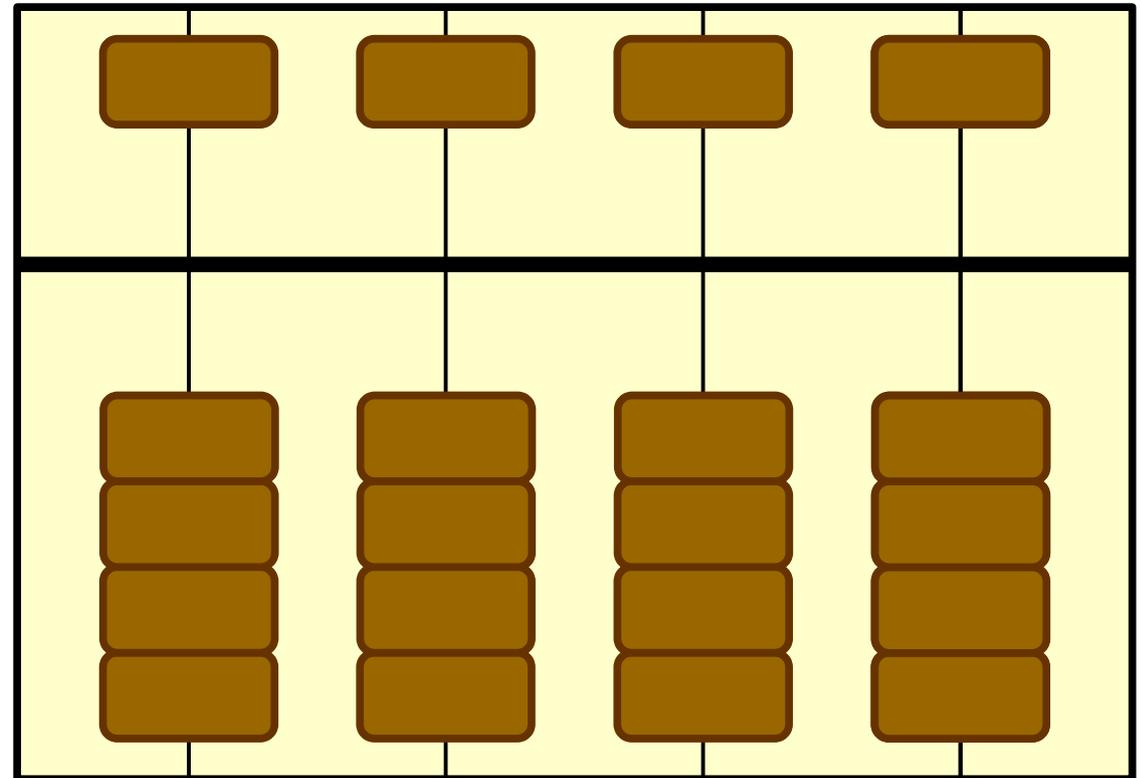
Calculators vs. Desktops

- A calculator has a ***small amount of memory***. A desktop computer has a ***large amount of memory***.
- A calculator performs a ***fixed set of functions***. A desktop is ***reprogrammable*** and can run many different programs.
- These two distinctions account for much of the difference between “calculator-like” computers and “desktop-esque” computers.
- In CS103, we’ll first explore “small-memory” computers in detail, then discuss “large-memory” computers in depth.

Our Goal: A Unifying Abstraction

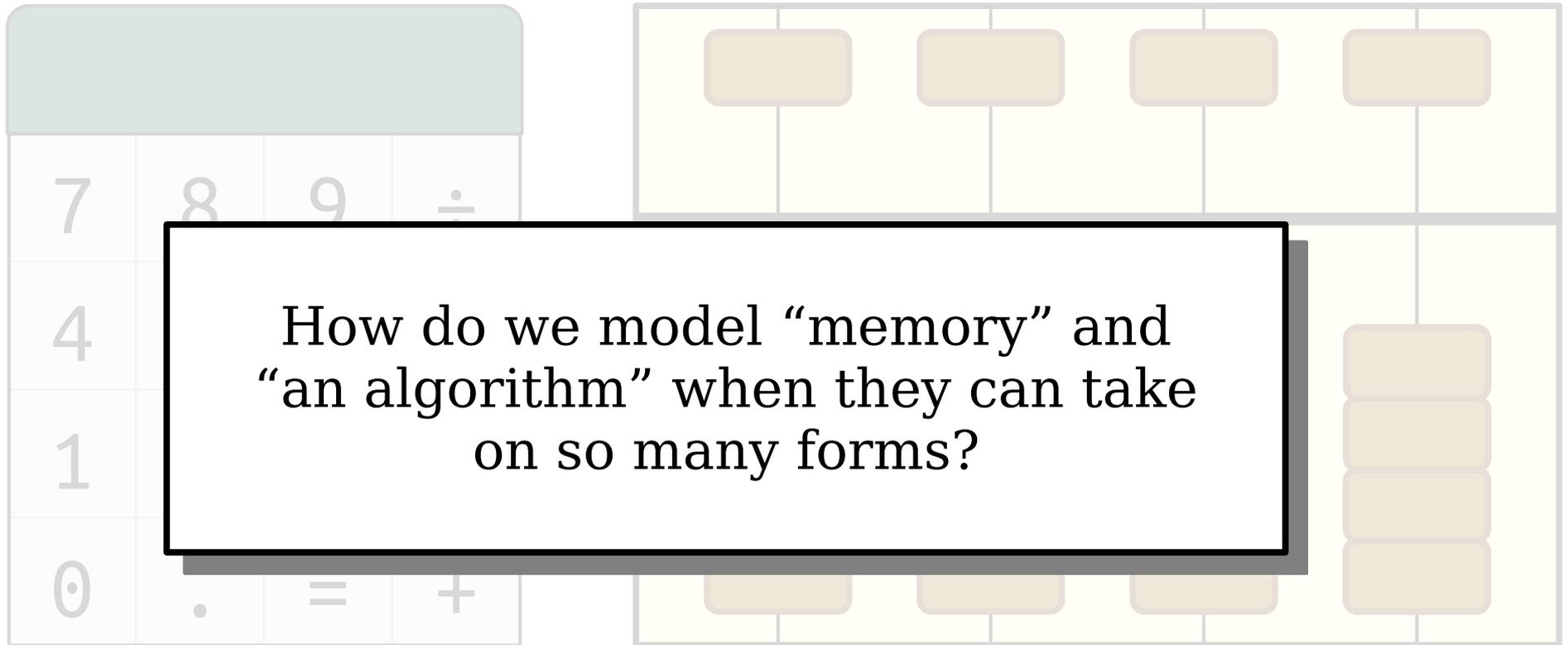
7	8	9	÷
4	5	6	×
1	2	3	-
0	.	=	+

Data stored electronically.
Algorithm is in silicon.
Memory limited by display.



Data stored in wood.
Algorithm is in brain.
Memory limited by beads.

Our Goal: A Unifying Abstraction



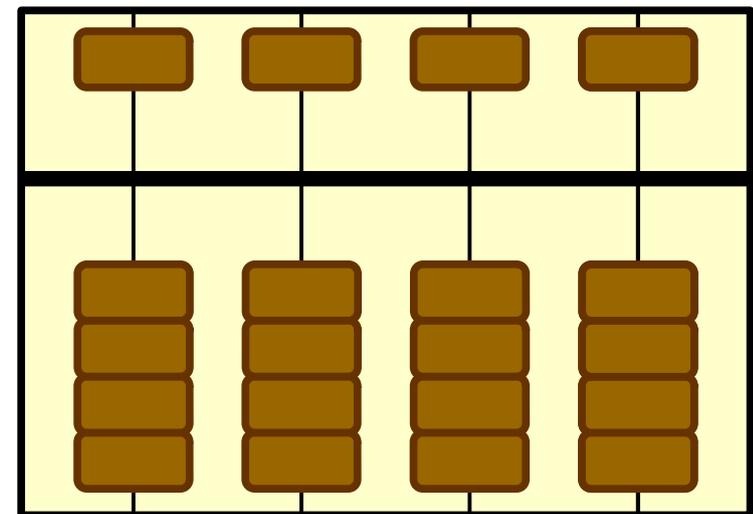
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What's in Common?

- These machines **receive input** from an external source.
- That input is provided **sequentially**, one discrete unit at a time.
- Each input causes the device to **change configuration**. This change, big or small, is where the computation happens.

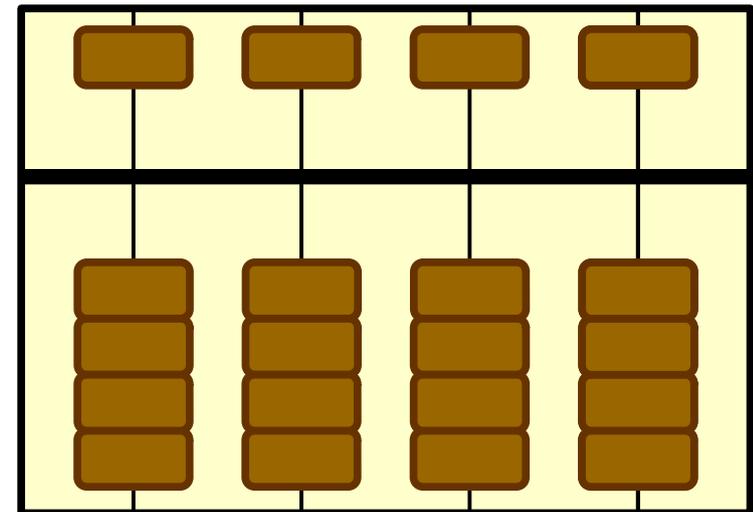
Calculator Keypad			
7	8	9	÷
4	5	6	×
1	2	3	-
0	.	=	+



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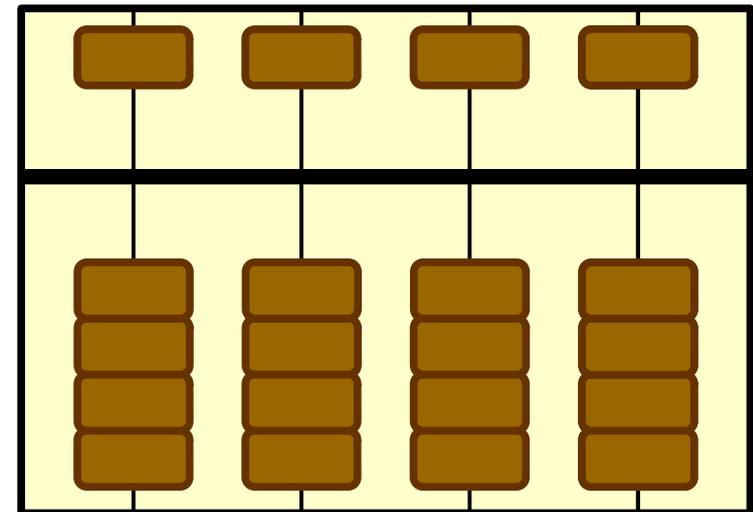
1			
7	8	9	÷
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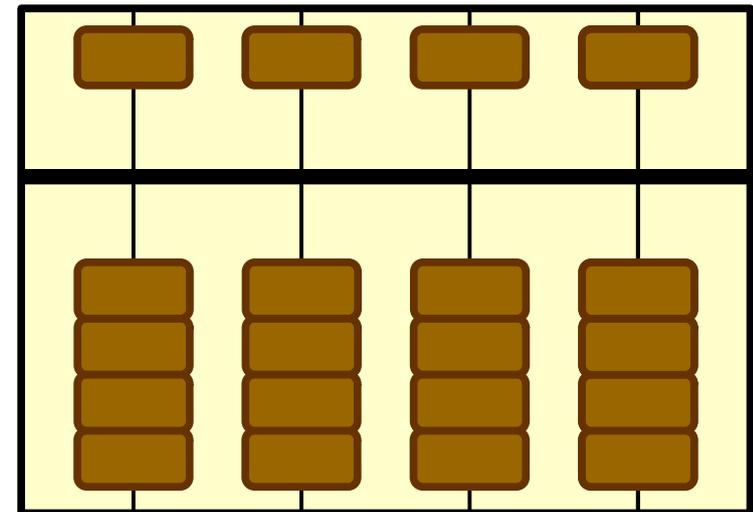
13			
7	8	9	÷
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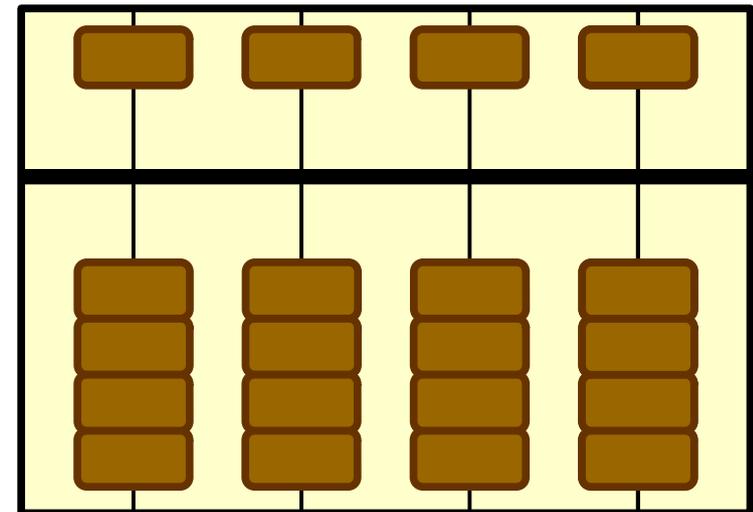
137			
7	8	9	÷
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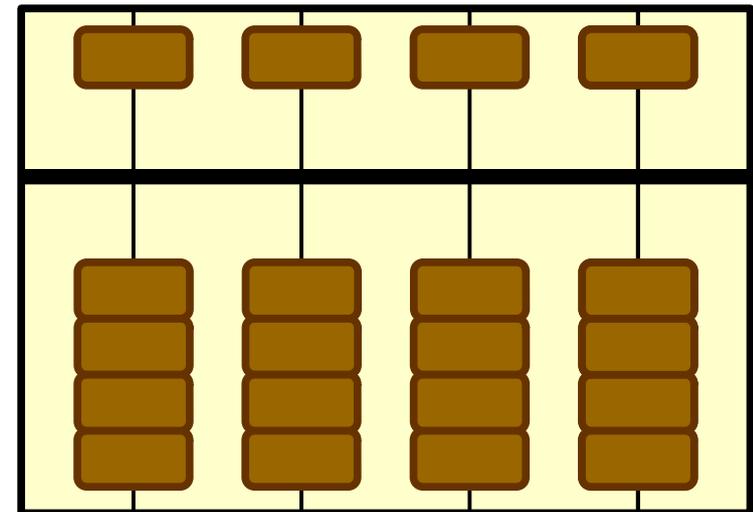
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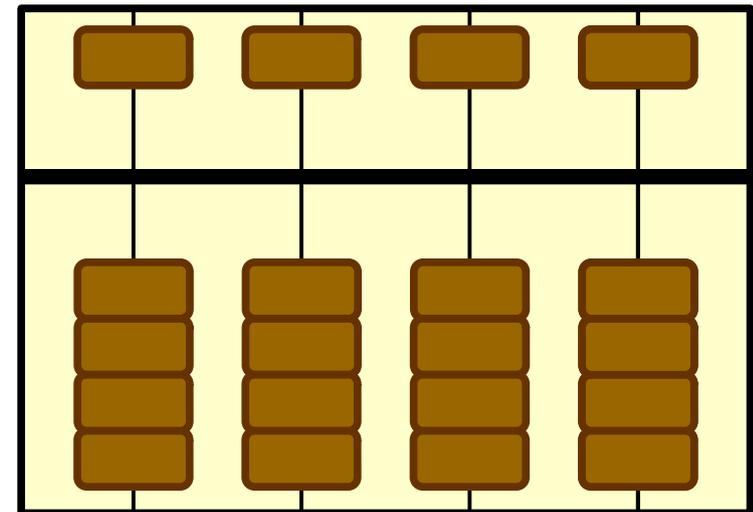
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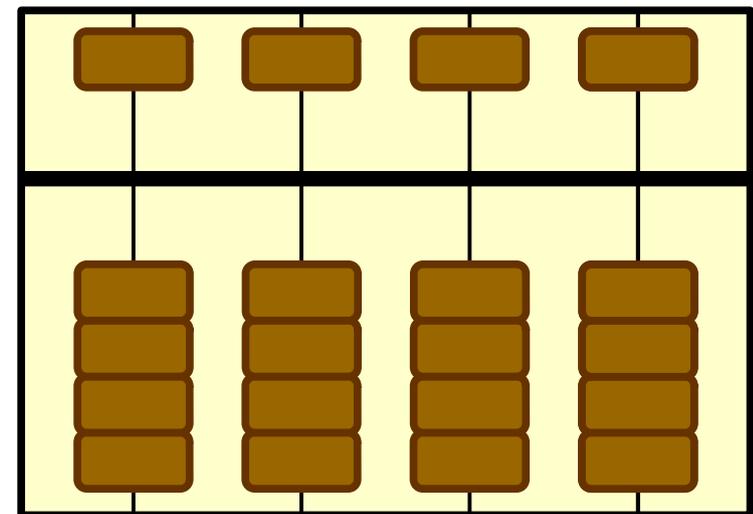
42			
7	8	9	÷
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0	.	=	+



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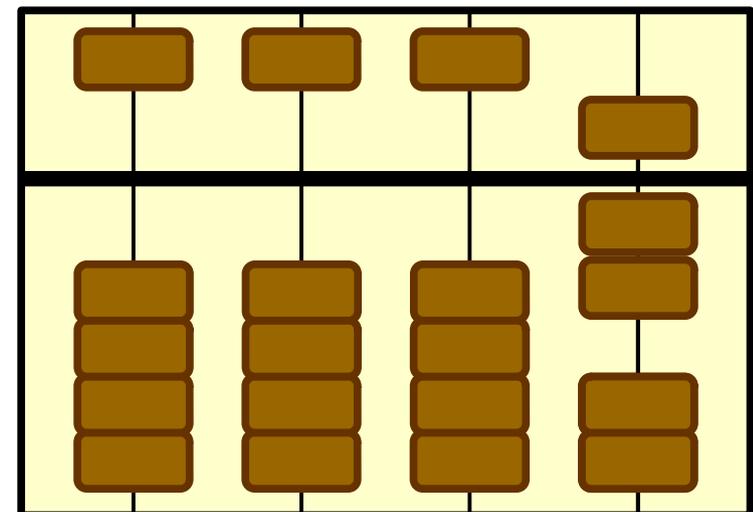
179			
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0	.	=	+



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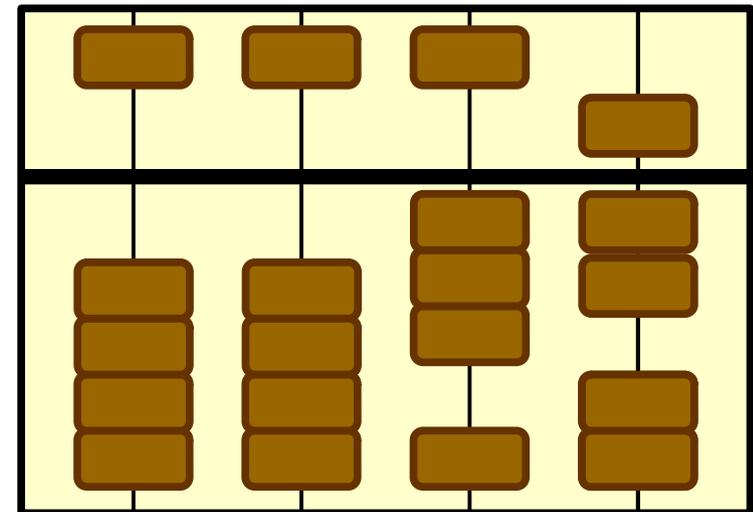
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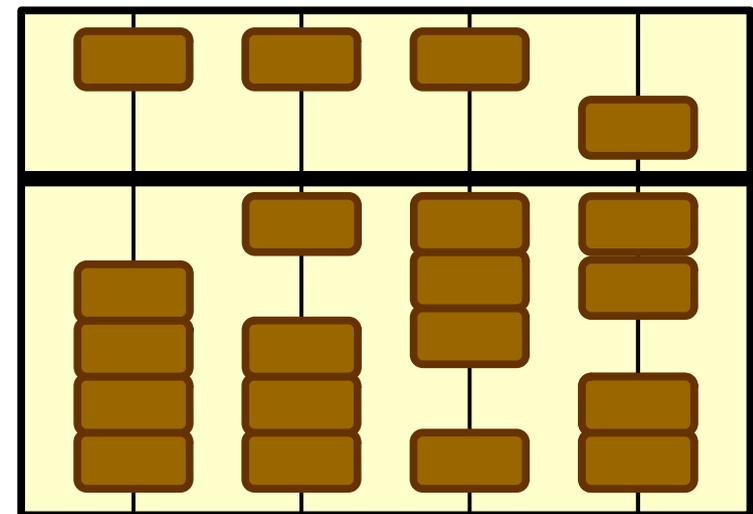
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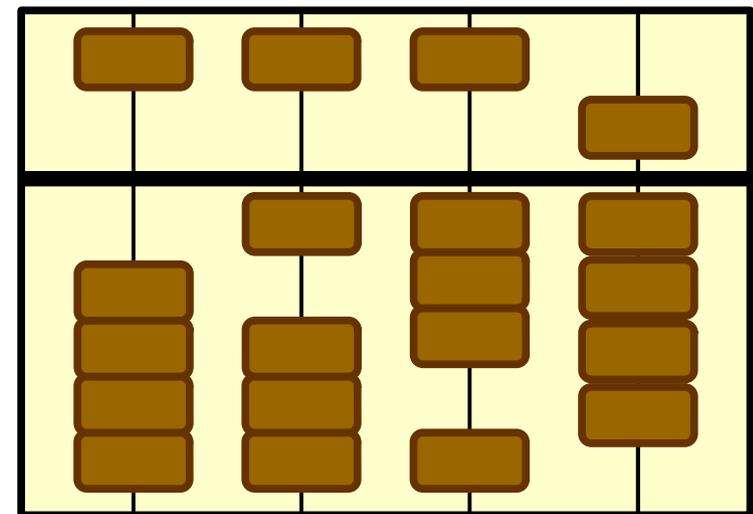
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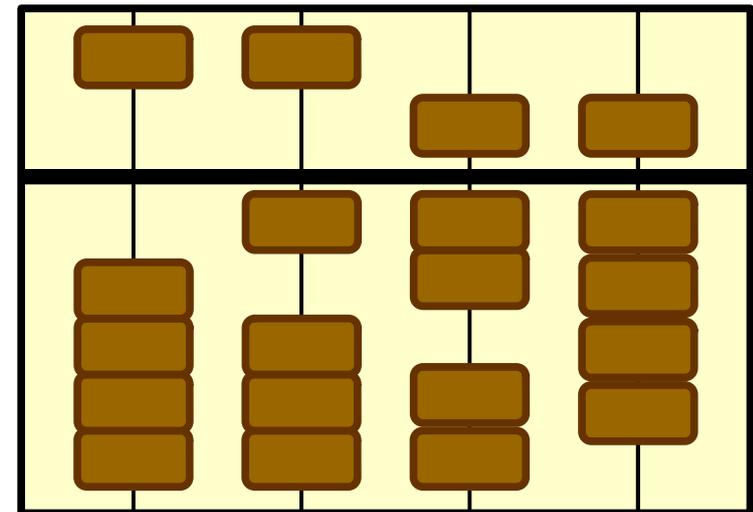
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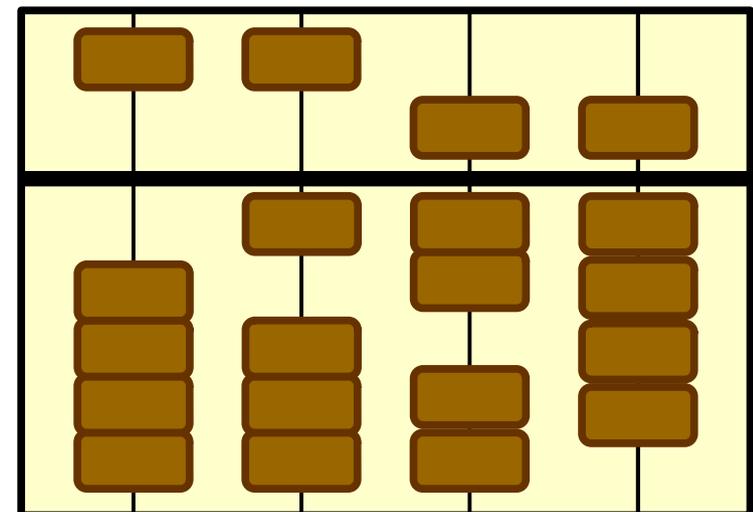
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- Each input causes the device to **change configuration**. This change, big or small, is where the computation happens.
- Once all input is provided, we can **read off an answer** based on the configuration of the device.

179			
7	8	9	÷
4	5	6	×
1	2	3	-
0	.	=	+



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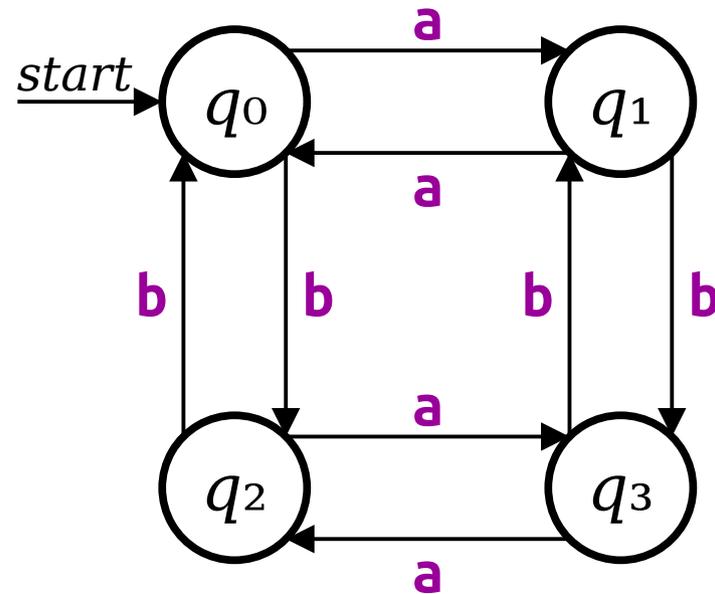
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Enter Automata

- An **automaton** (plural: **automata**) is a mathematical model of a computing device.
- It's an **abstraction** of a real computer, the way that graphs are abstractions of social networks, transportation grids, etc.
- The automata we'll explore are
 - powerful enough to capture huge classes of computing devices, yet
 - simple enough that we can reason about them in a small space.
- They're also fascinating and useful in their own rights. More on that later.

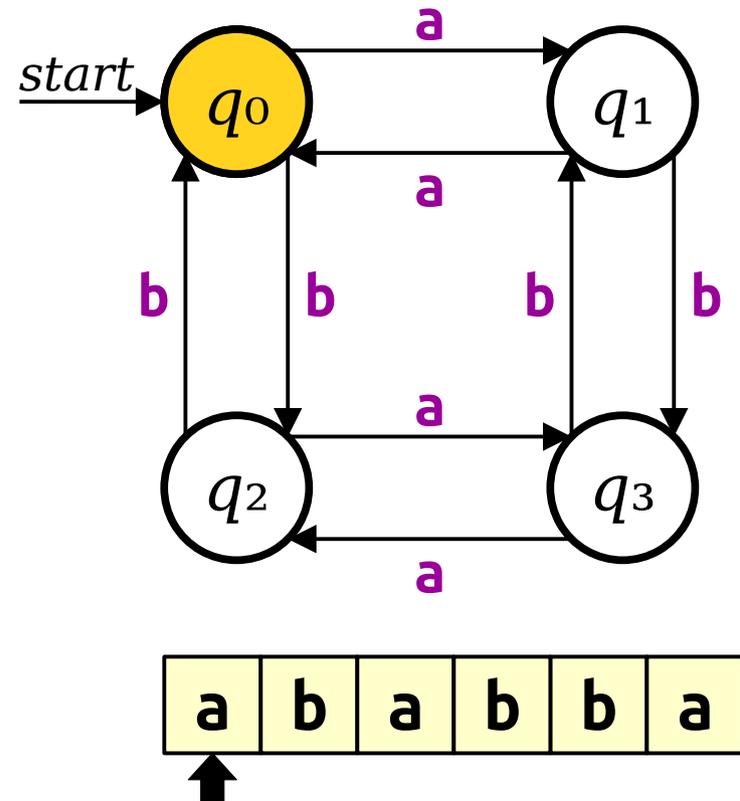
Modeling Finite Computation

- We will model a finite-memory computer as a collection of **states** linked by **transitions**.
- Each state corresponds to one possible configuration of the device's memory. This is super abstract!
- Each transition indicates how memory changes in response to inputs.
- Some state is designated as the **start state**. The computation begins in that state.



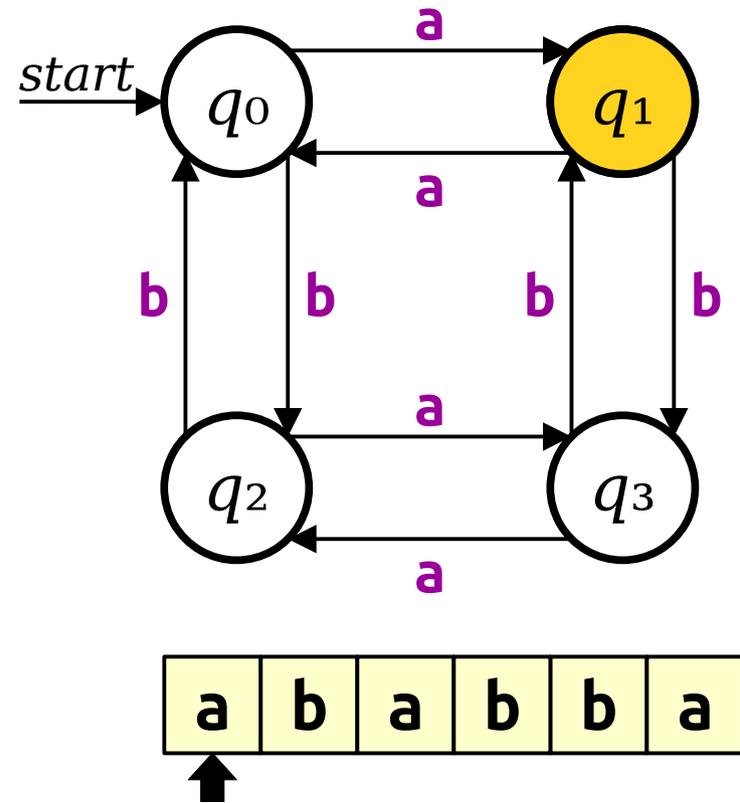
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 - Each character represents some external input to the device.
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- To run this device, we begin in our start state and scan the input from left to right.
- Each time the machine sees a character, it *changes state* by following the transition labeled with that character.



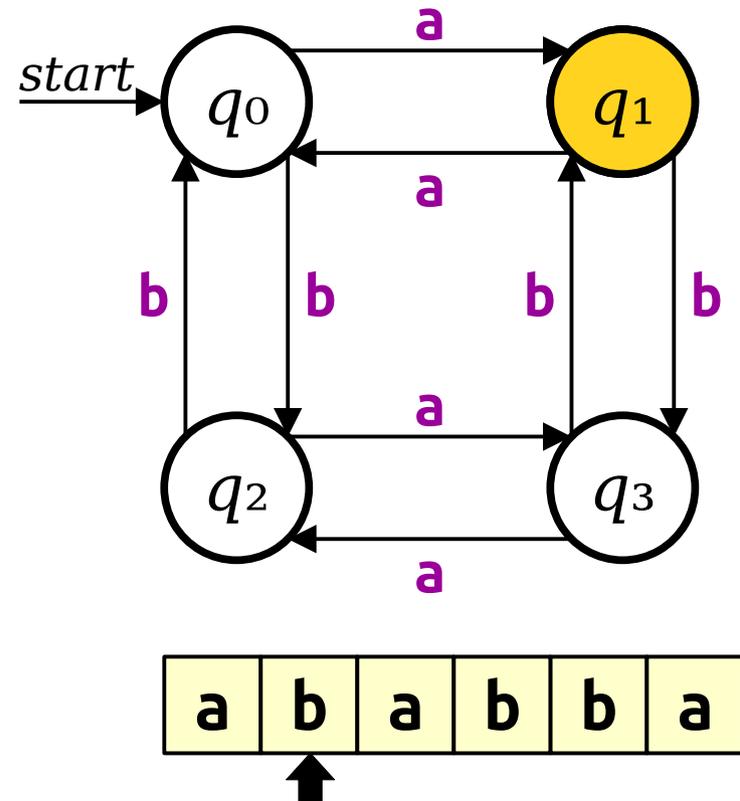
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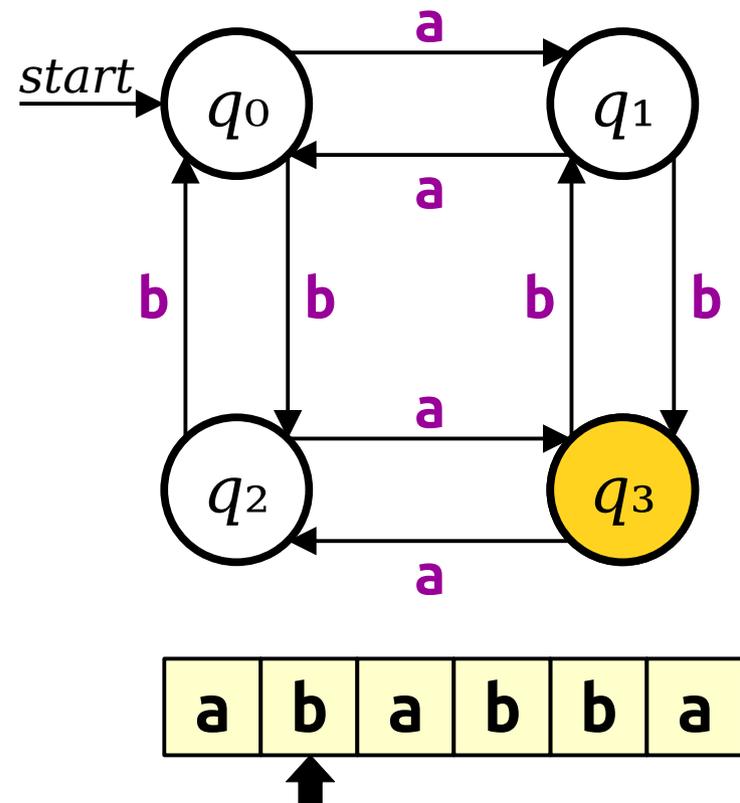
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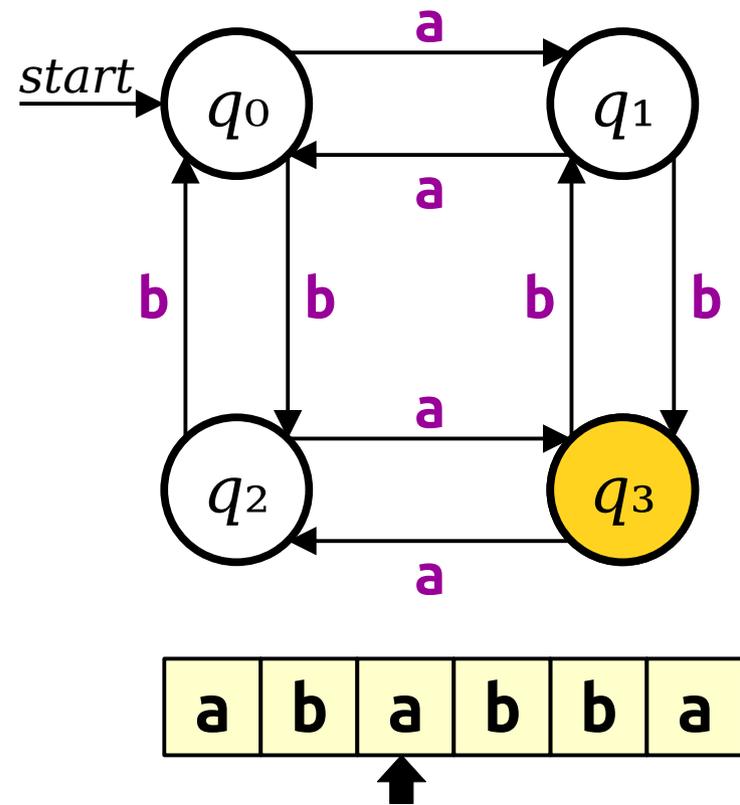
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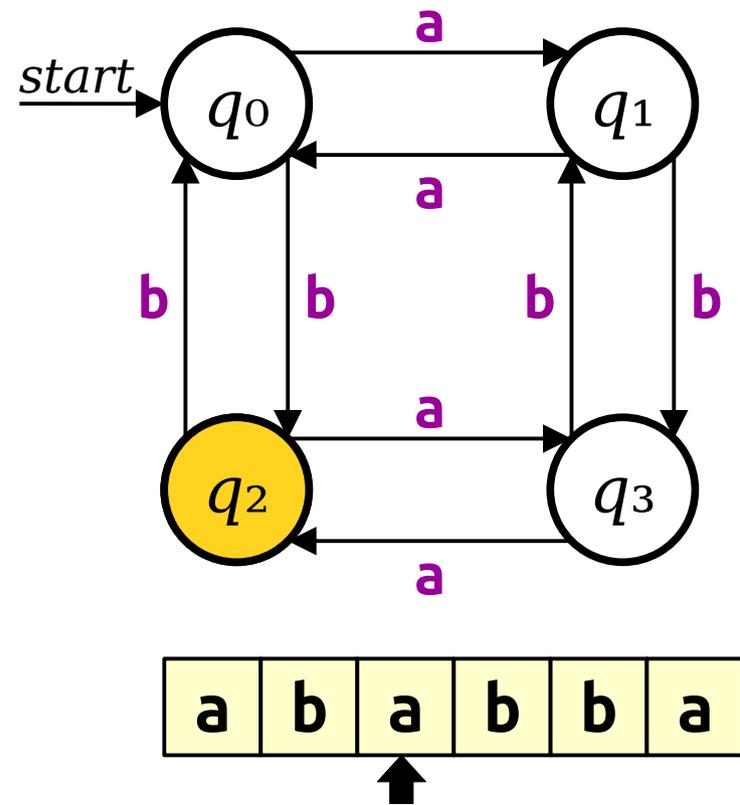
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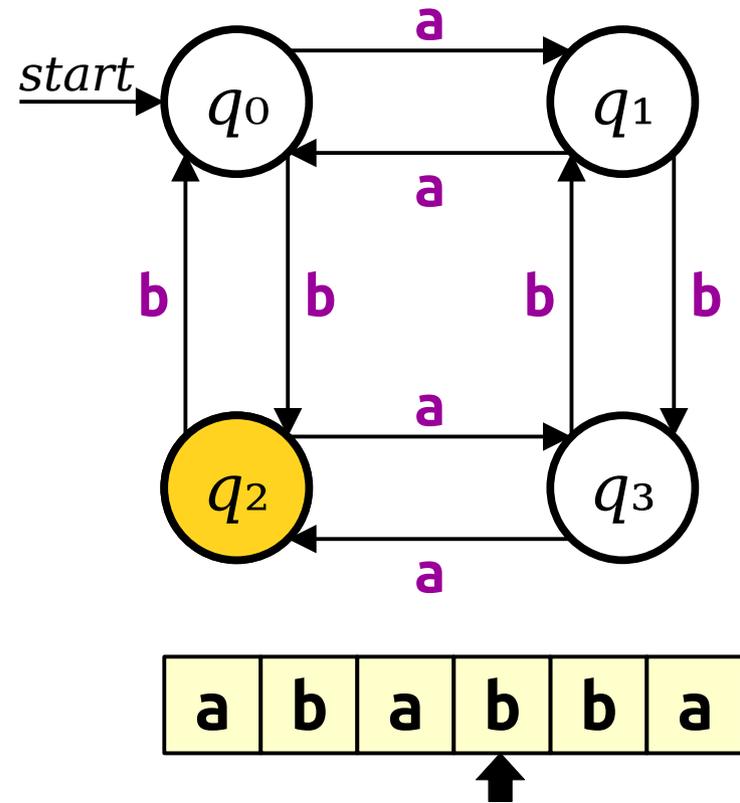
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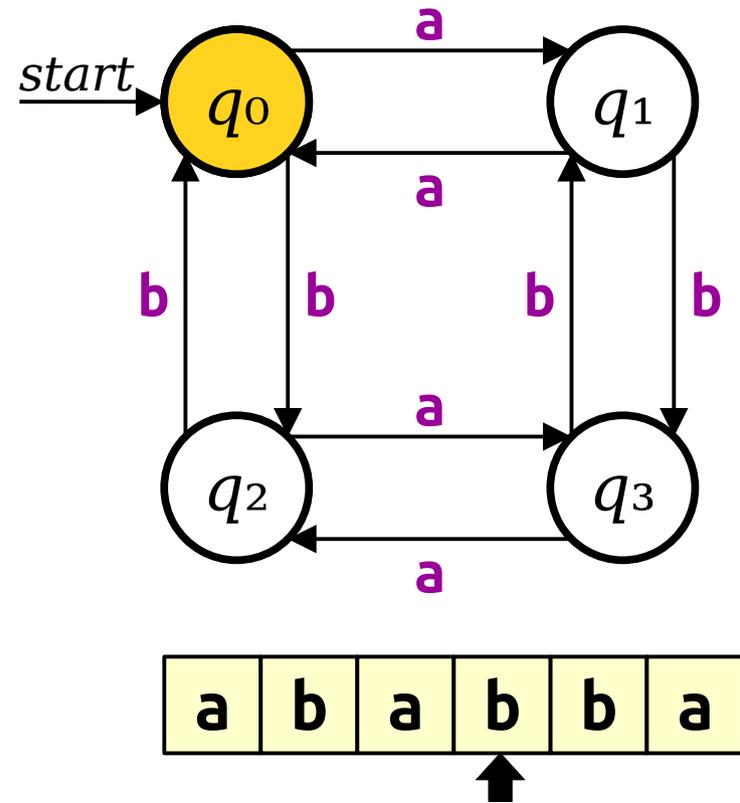
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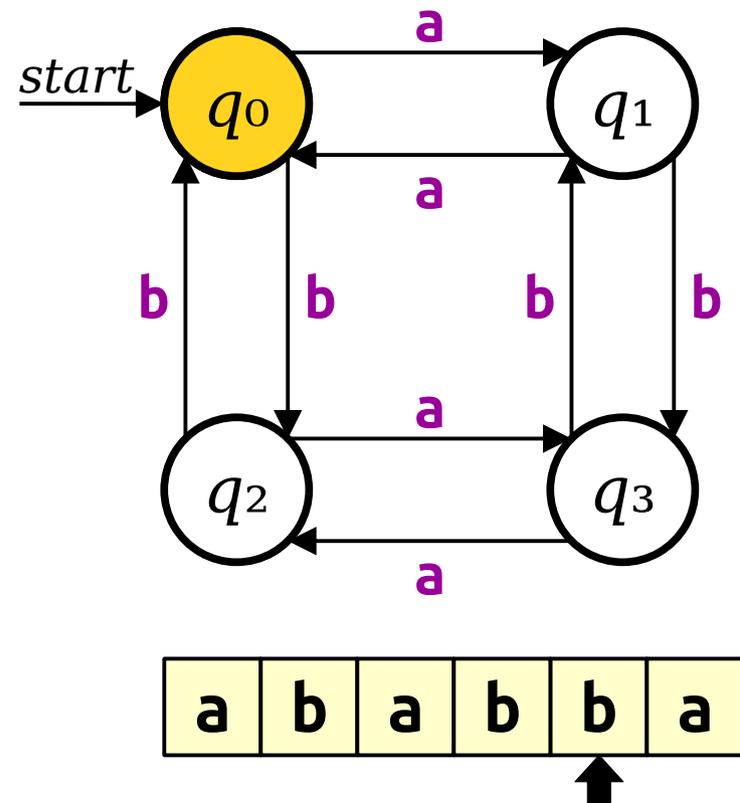
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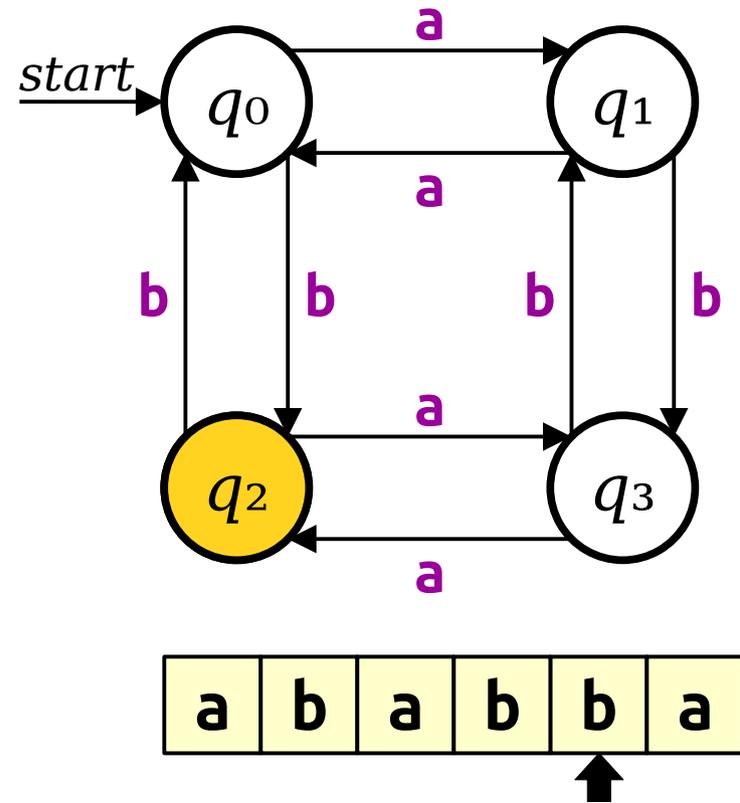
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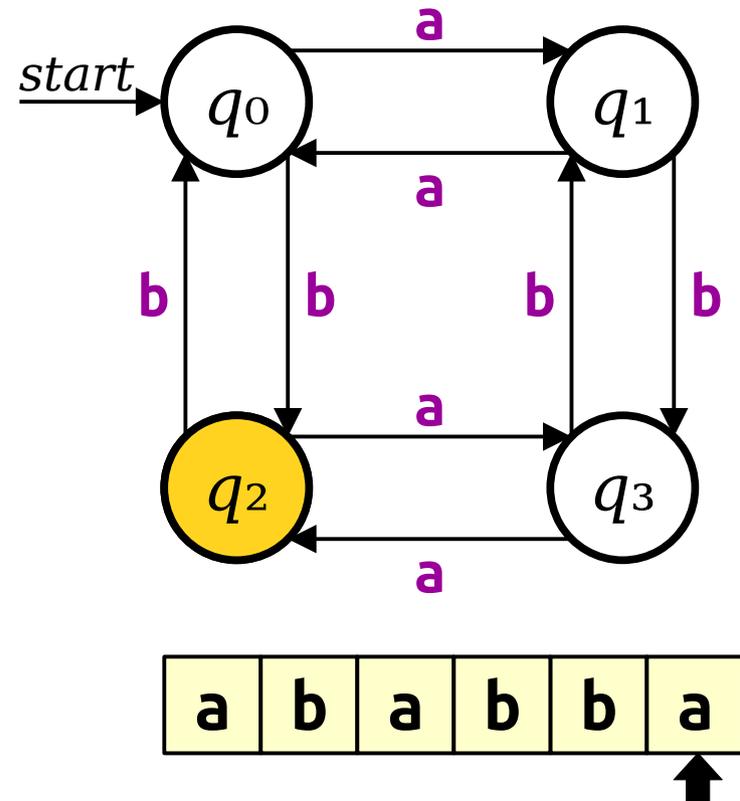
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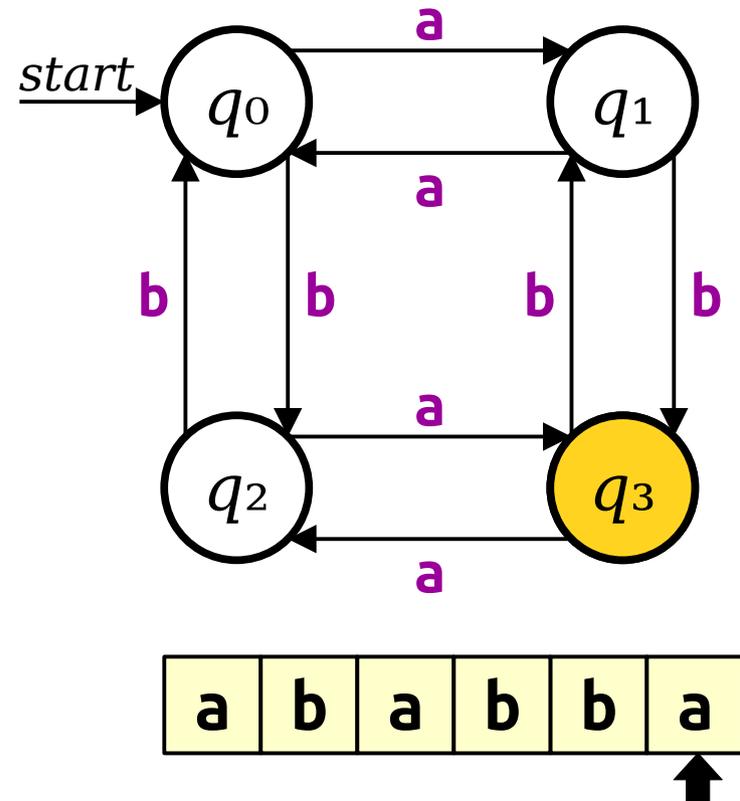
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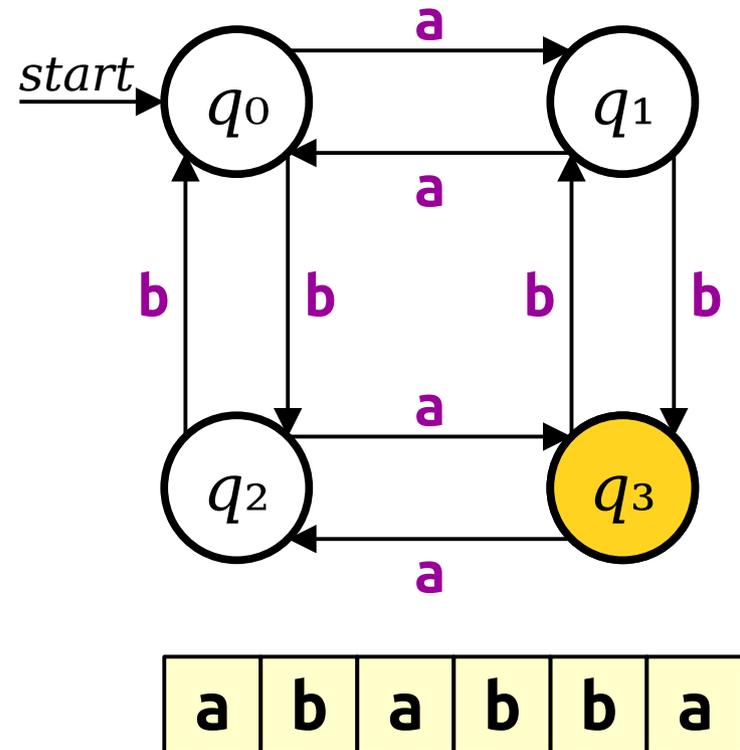
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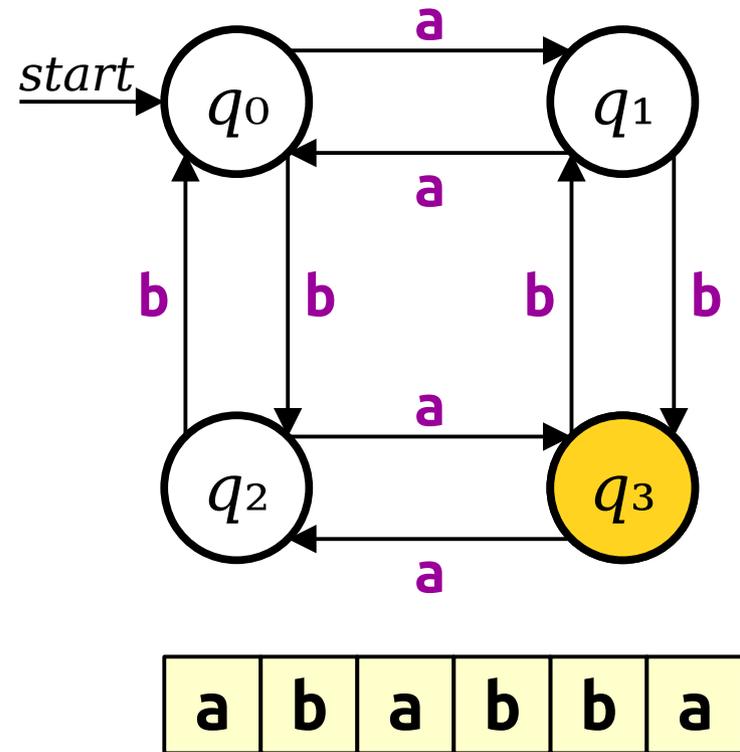
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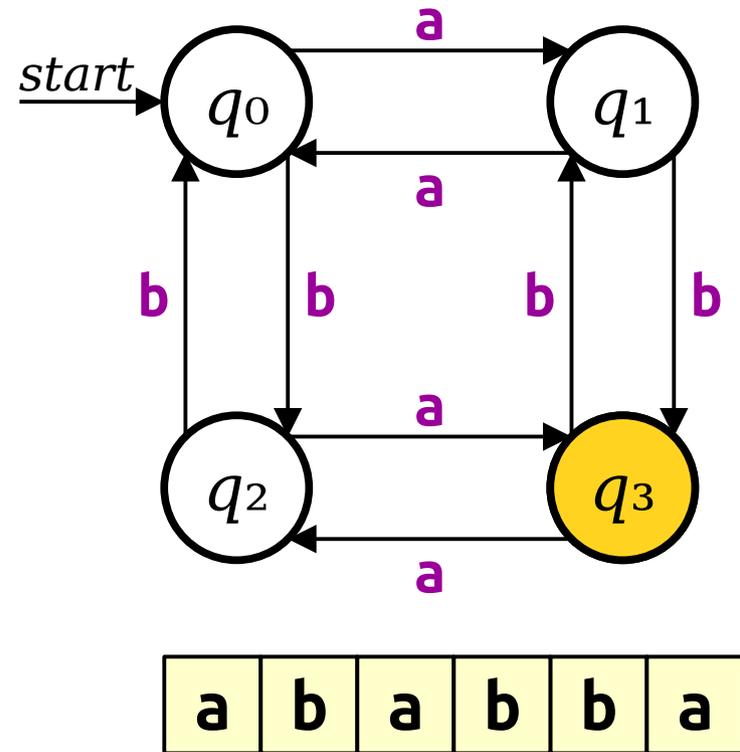
Modeling Finite Computation

- Once we've finished entering all the characters of our input, we need to obtain the result of the computation.
- In general, computers can produce all sorts of things as the result of a computation: a number, a piece of text, etc.
- As a simplifying assumption, we'll assume that we just need to get a single bit of output. That is, our machines will just say YES or NO.
- (This can be generalized - come talk to us after class if you're curious how!)



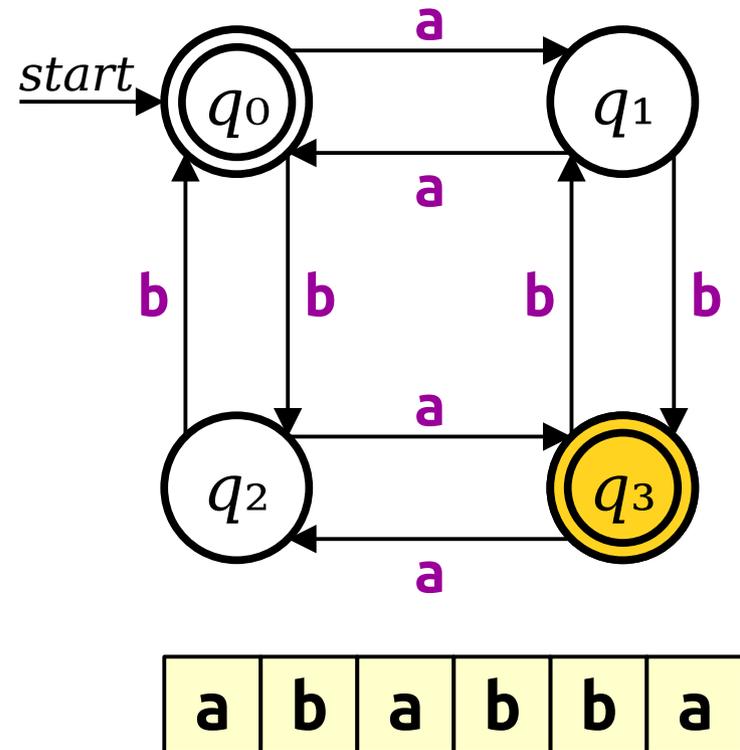
Modeling Finite Computation

- Some of the states in our computational device will be marked as **accepting states**. These are denoted with a double ring.



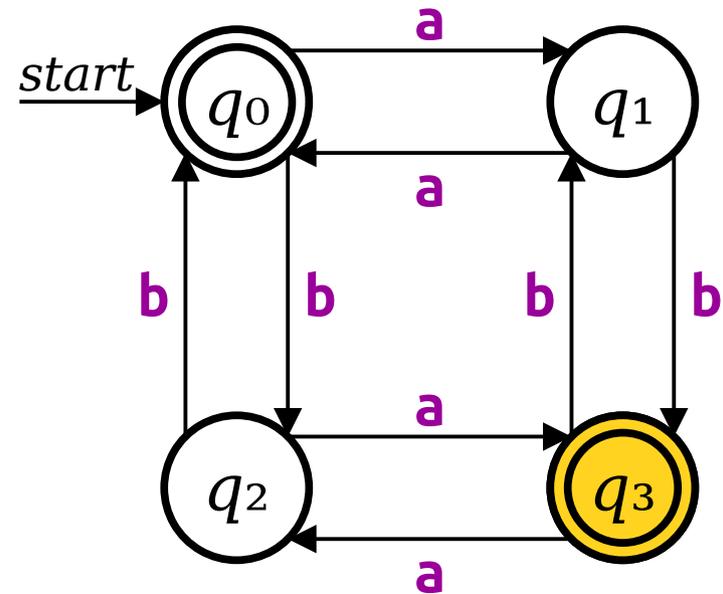
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- Some of the states in our computational device will be marked as **accepting states**. These are denoted with a double ring.
- If the device ends in an accepting state after seeing all the input, **accepts** the input (says YES)
- If the device does not end in an accepting state after seeing all the input, it **rejects** the input (says NO).



Modeling Finite Computation

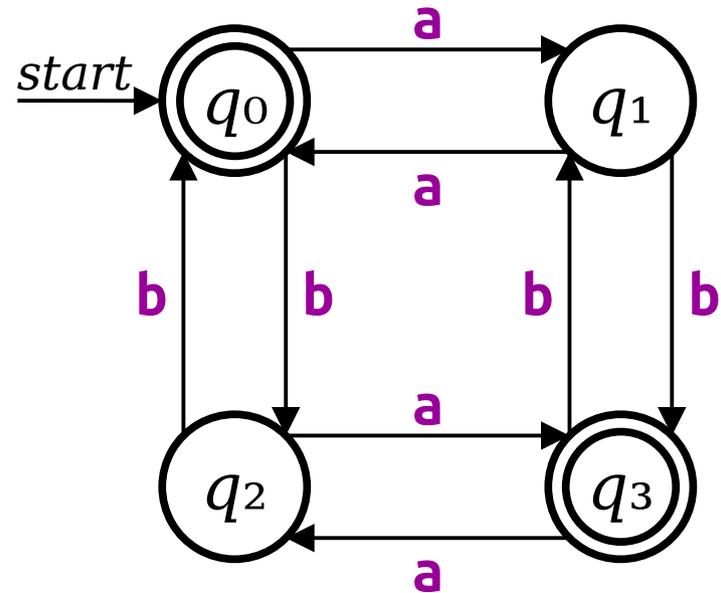
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Modeling Finite Computation

- Try it yourself! Which of these strings does this device accept?

aab
aabb
abbababba



Answer at

<https://cs103.stanford.edu/pollev>

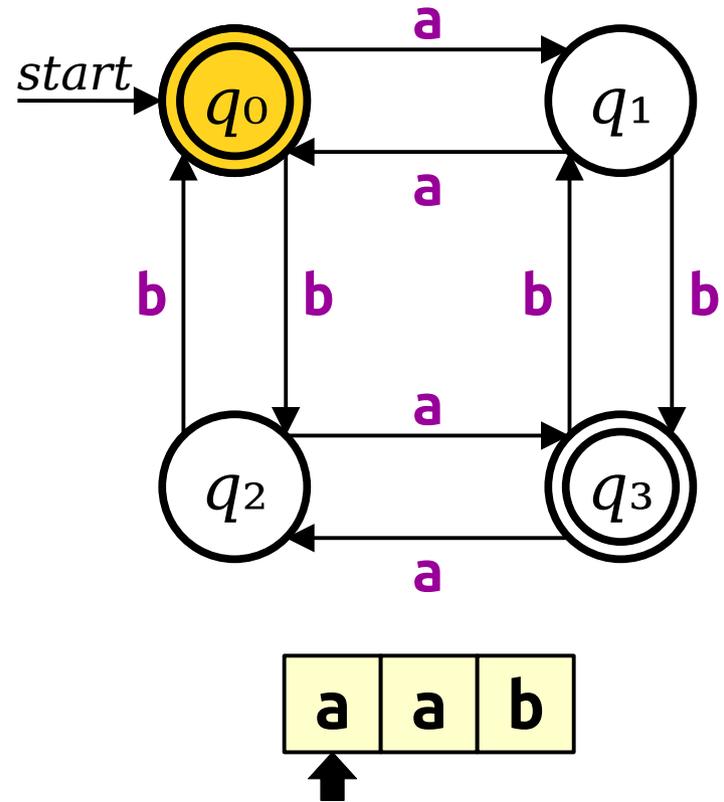
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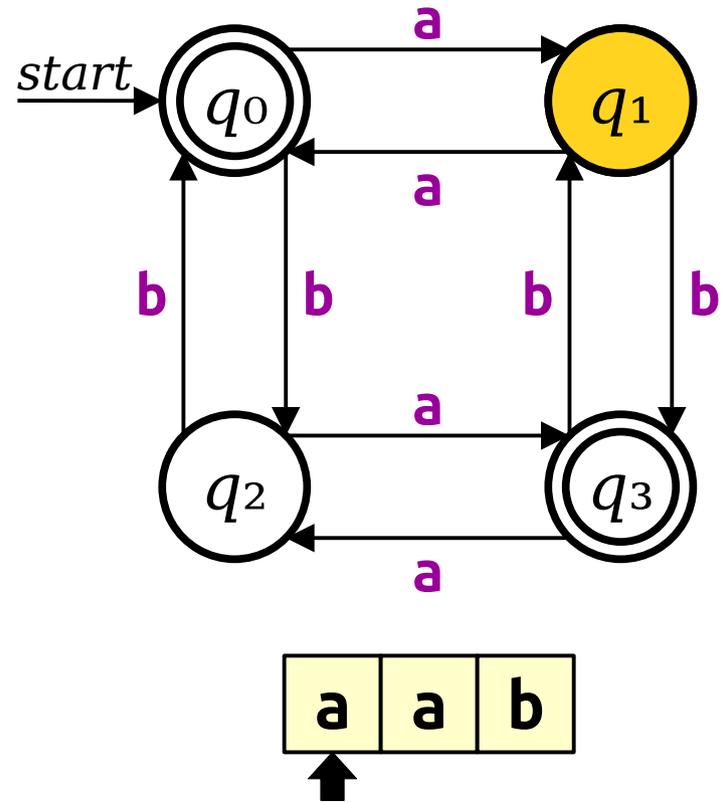
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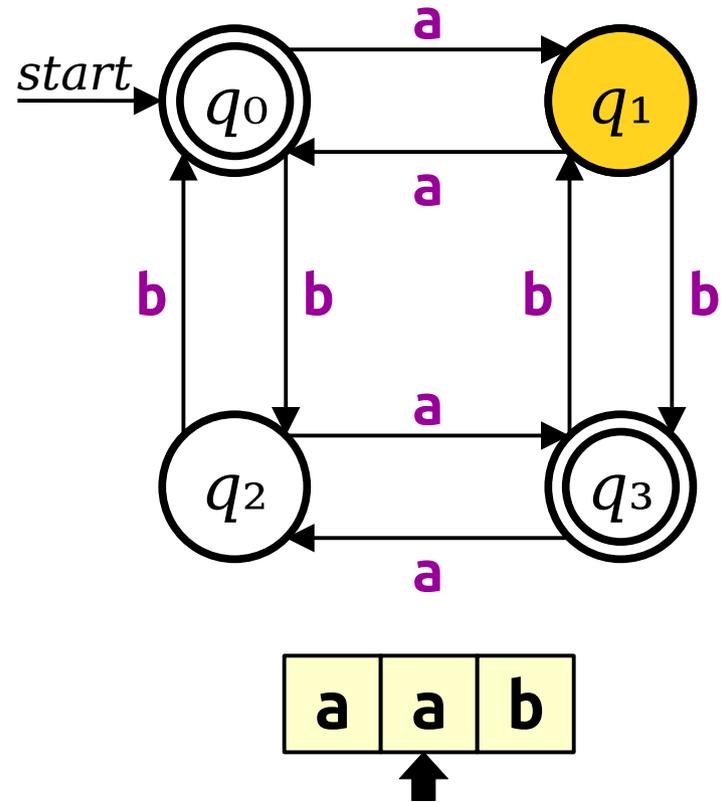
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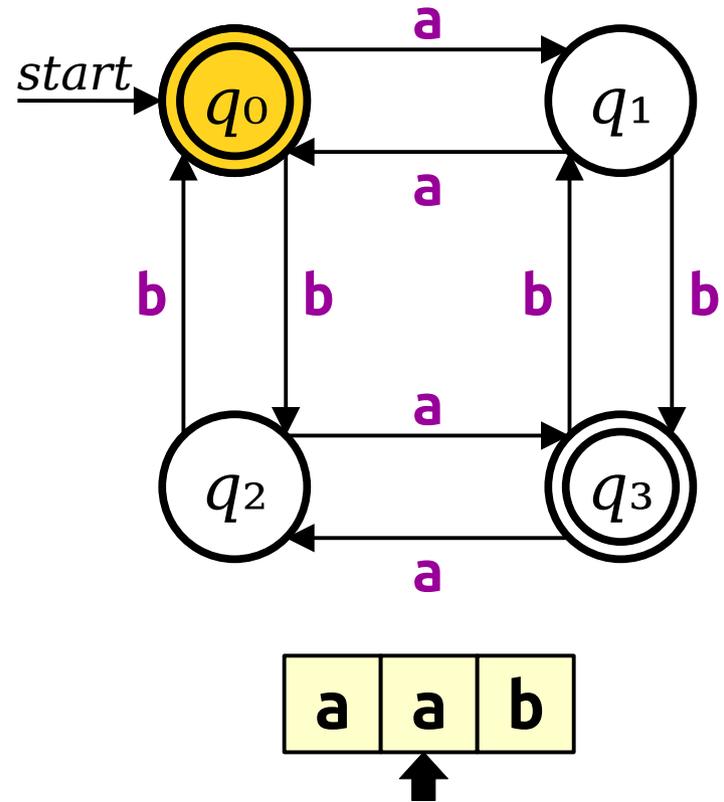
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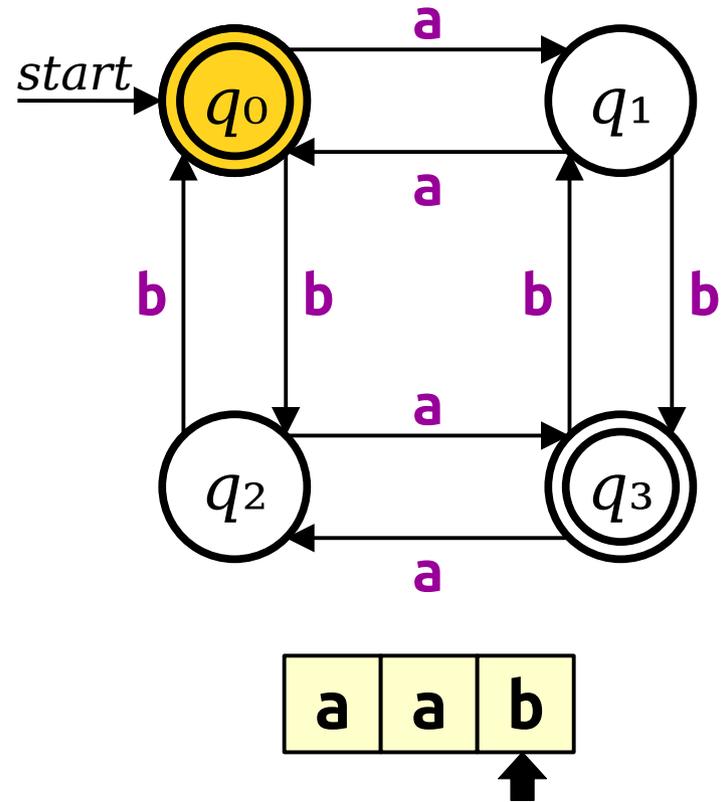
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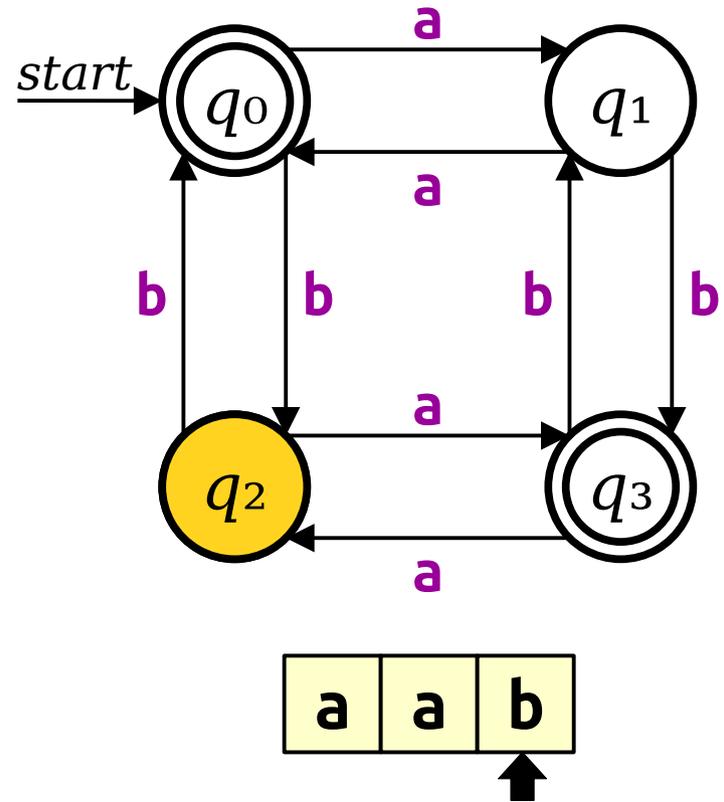
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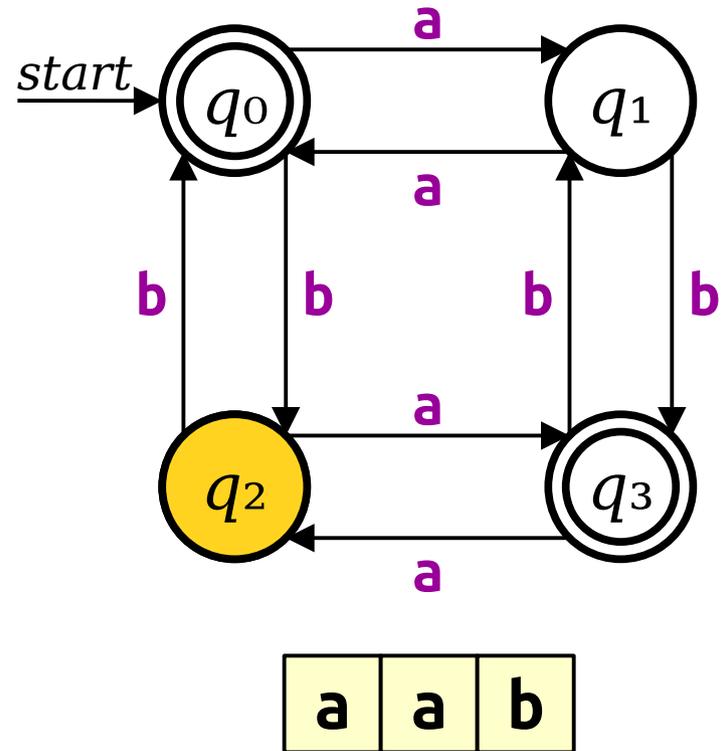
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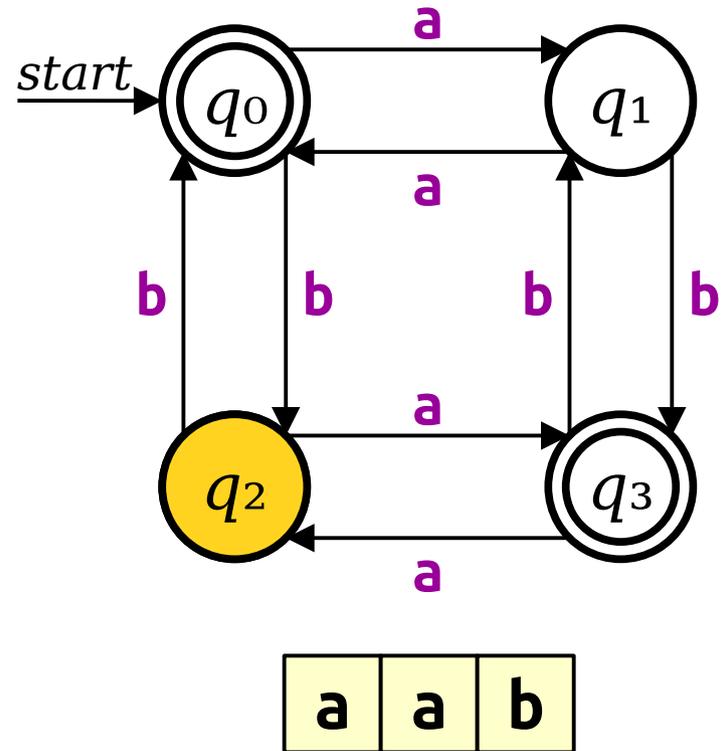
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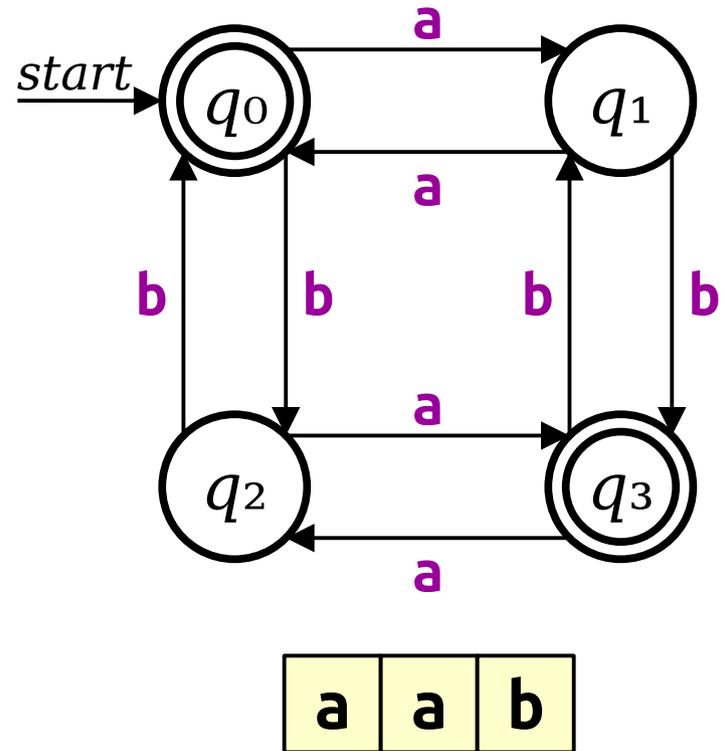
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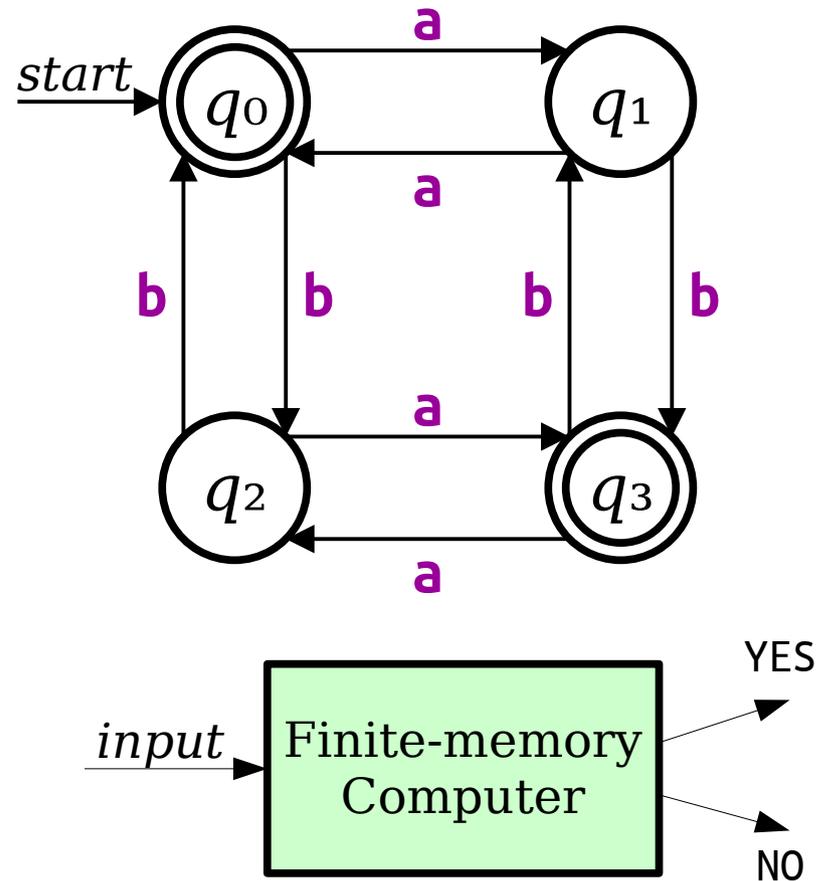
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abbababba



Finite Automata

- This type of computational device is called a **finite automaton** (plural: **finite automata**).
- Finite automata model computers where (1) memory is finite and (2) the computation produces as YES/NO answer.
- In other words, finite automata model predicates, and do so with a fixed, finite amount of memory.



Finite Automata

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Formalizing Things

Strings

- An **alphabet** is a finite, nonempty set of symbols called **characters**.
 - Typically, we use the symbol Σ to refer to an alphabet.
- A **string over an alphabet Σ** is a finite sequence of characters drawn from Σ .
- Example: Let $\Sigma = \{a, b\}$. Here are some strings over Σ :
a aabaaabbabaaabaaaabbb abbababba
- But wait! There are no quotes here!
- The **empty string** has no characters and is denoted ϵ .

Languages

- A **language over Σ** is a set L consisting of strings over Σ .
- Example: The language of palindromes over $\Sigma = \{a, b, c\}$ is the set
 - $\{\varepsilon, a, b, c, aa, bb, cc, aaa, aba, aca, bab, \dots\}$
- The set of all strings composed from letters in Σ is denoted Σ^* .
 - Formally: $\Sigma^* = \{ w \mid w \text{ is a string over } \Sigma \}$.
- Formally, we say that L is a language over Σ when $L \subseteq \Sigma^*$.

Mathematical Lookalikes

- We now have \in , ε , Σ , and Σ^* . Yikes!
- The symbol \in is the... ?

Mathematical Lookalikes

- We now have \in , ε , Σ , and Σ^* . Yikes!
- The symbol \in is the ***element-of*** relation.

Mathematical Lookalikes

- We now have \in , ε , Σ , and Σ^* . Yikes!
- The symbol \in is the *element-of* relation.
- The symbol ε is the... ?

Mathematical Lookalikes

- We now have \in , ε , Σ , and Σ^* . Yikes!
- The symbol \in is the ***element-of*** relation.
- The symbol ε is the ***empty string***.

Mathematical Lookalikes

- We now have \in , ε , Σ , and Σ^* . Yikes!
- The symbol \in is the ***element-of*** relation.
- The symbol ε is the ***empty string***.
- The symbol Σ denotes... ?

Mathematical Lookalikes

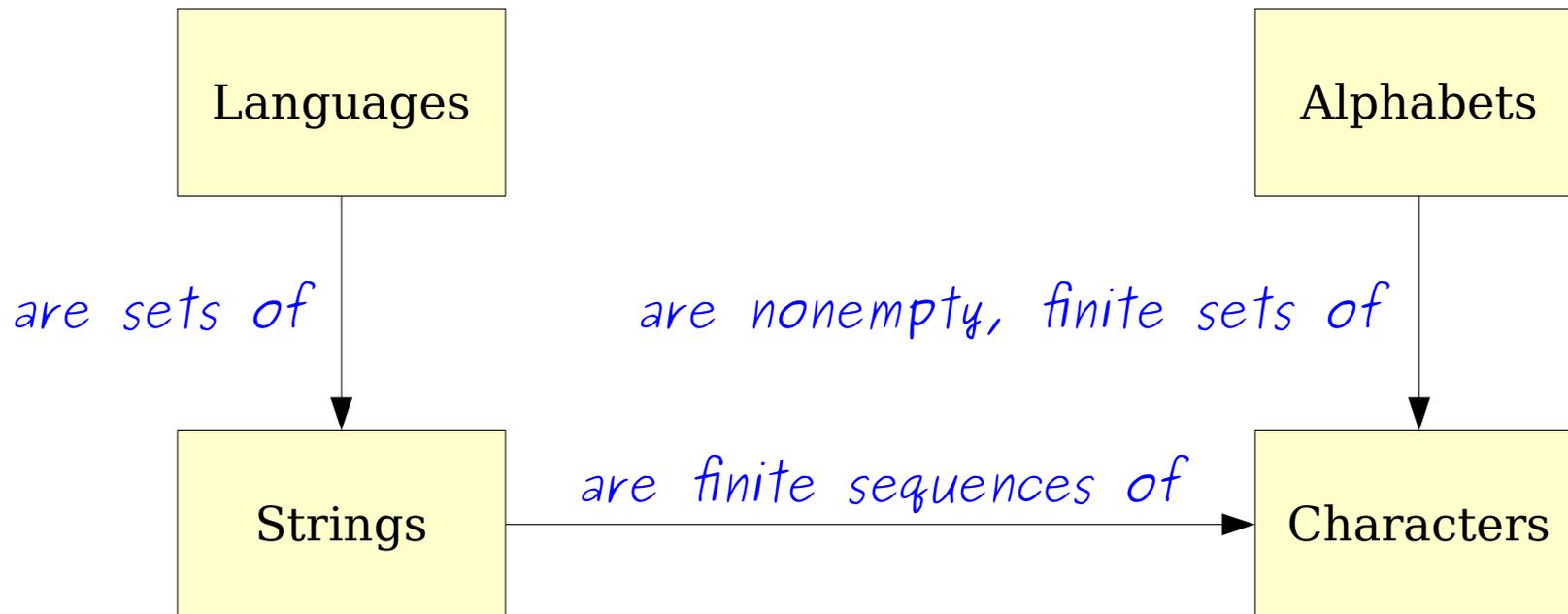
- We now have \in , ε , Σ , and Σ^* . Yikes!
- The symbol \in is the ***element-of*** relation.
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Mathematical Lookalikes

- We now have \in , ε , Σ , and Σ^* . Yikes!
- The symbol \in is the ***element-of*** relation.
- The symbol ε is the ***empty string***.
- The symbol Σ denotes an ***alphabet***.
- The expression Σ^* means “all strings that can be made from characters in Σ .”
- That lets us write things like
 - We have $\varepsilon \in \Sigma^*$, but $\varepsilon \notin \Sigma$.
- Ever get confused? ***Just ask!***

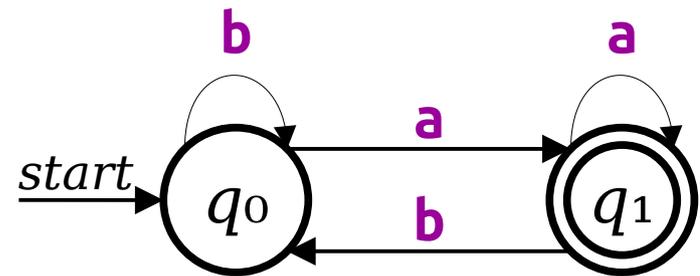
The Cast of Characters

- **Languages** are sets of strings.
- **Strings** are finite sequences of characters.
- **Characters** are individual symbols.
- **Alphabets** are sets of characters.



Finite Automata and Languages

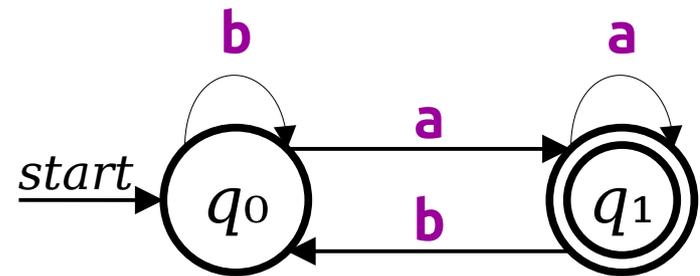
- Let A be an automaton that processes strings drawn from an alphabet Σ .
- The **language of A** , denoted $\mathcal{L}(A)$, is the set of strings over Σ that A accepts:



$$\mathcal{L}(A) = \{ w \in \Sigma^* \mid A \text{ accepts } w \}$$

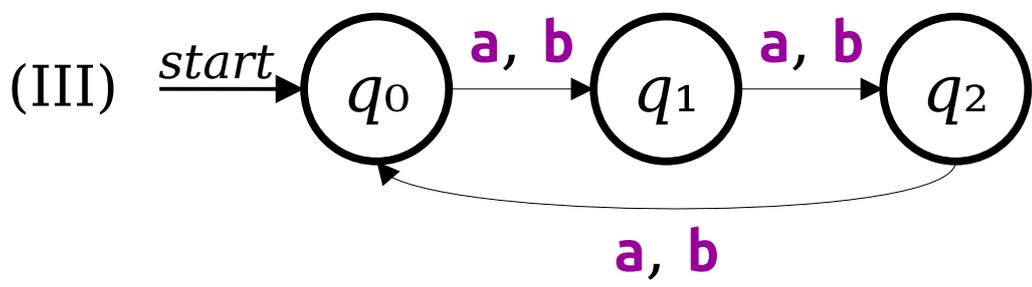
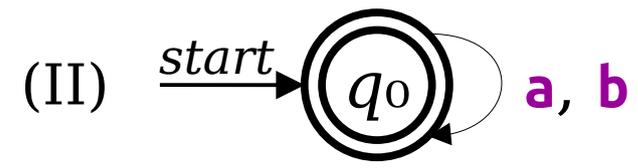
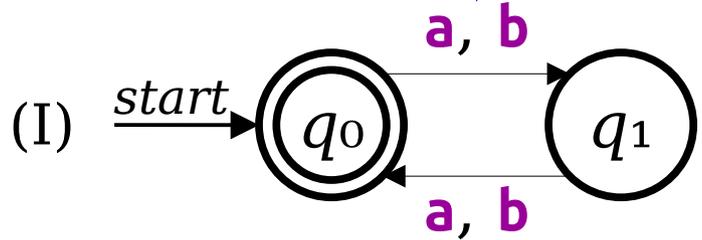
Finite Automata and Languages

- Let D be the automaton shown to the right. It processes strings over $\{\mathbf{a}, \mathbf{b}\}$.
- Notice that D accepts all strings of \mathbf{a} 's and \mathbf{b} 's that end in \mathbf{a} and rejects everything else.
- So $\mathcal{L}(D) = \{ w \in \{\mathbf{a}, \mathbf{b}\}^* \mid w \text{ ends in } \mathbf{a} \}$.



$$\mathcal{L}(A) = \{ w \in \Sigma^* \mid A \text{ accepts } w \}$$

This means “take this transition if you see an **a** or a **b**.”



What are the languages of these automata? Answer at <https://cs103.stanford.edu/pollev>

$$\mathcal{L}(A) = \{ w \in \Sigma^* \mid A \text{ accepts } w \}$$

The Story So Far

- A ***finite automaton*** is a collection of ***states*** joined by ***transitions***.
- Some state is designated as the ***start state***.
- Some number of states are designated as ***accepting states***.
- The automaton processes a string by beginning in the start state and following the indicated transitions.
- If the automaton ends in an accepting state, it ***accepts*** the input.
- Otherwise, the automaton ***rejects*** the input.
- The ***language*** of an automaton is the... ?

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- The ***language*** of an automaton is the set of strings it accepts.

Finite Automata

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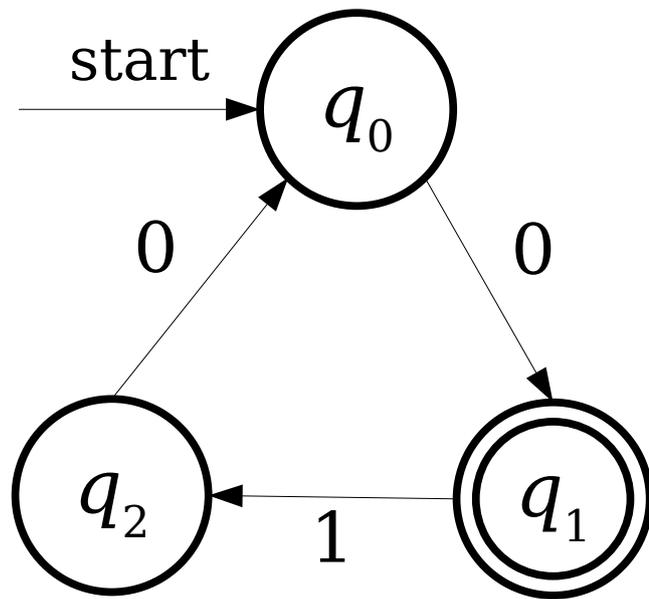
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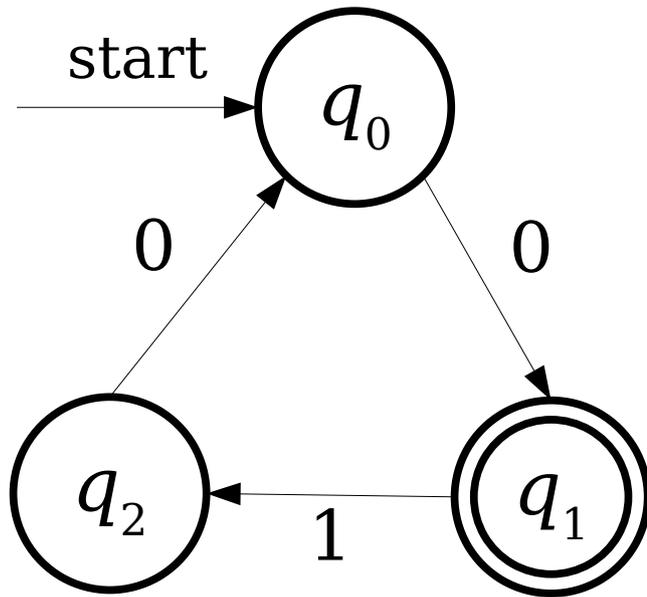
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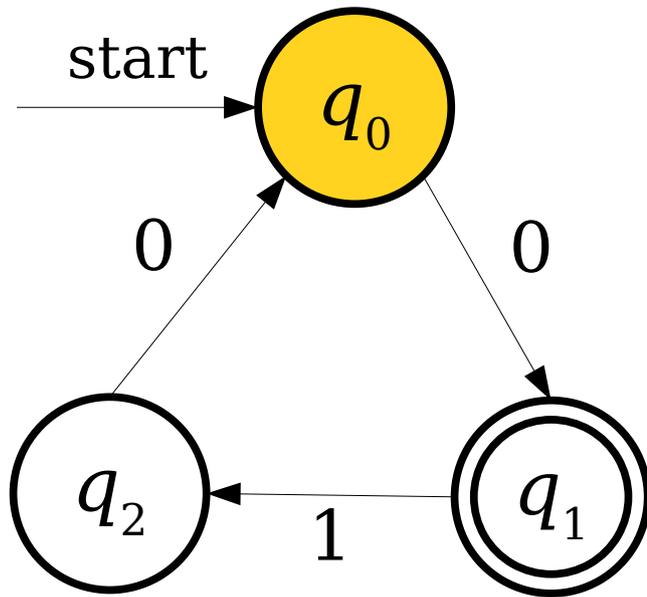
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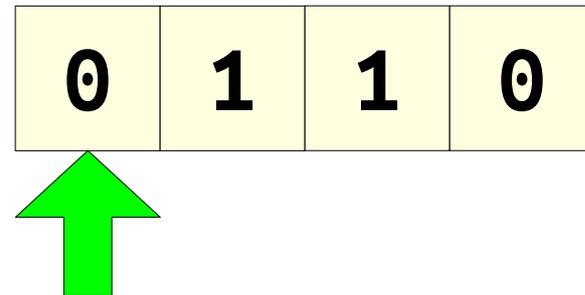
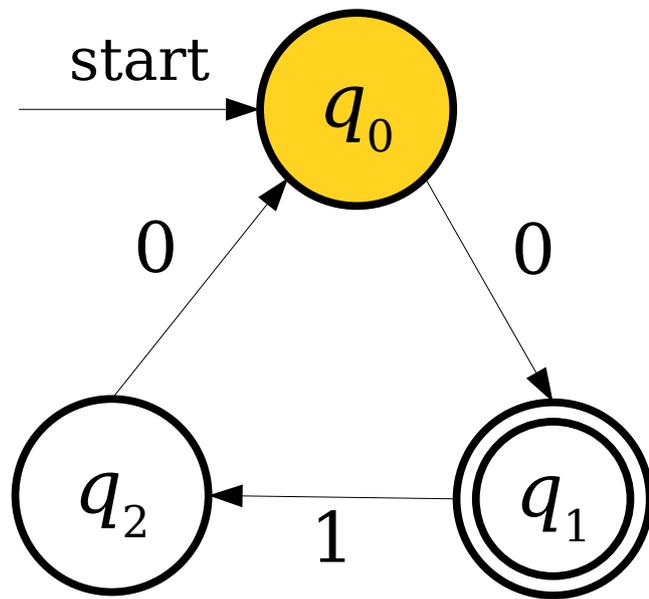
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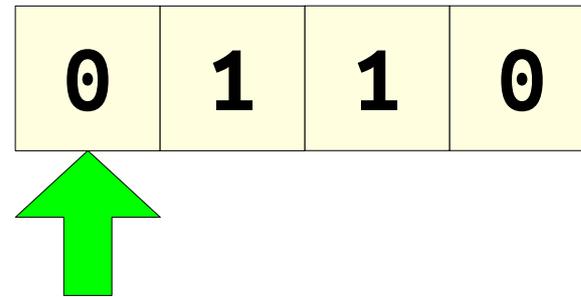
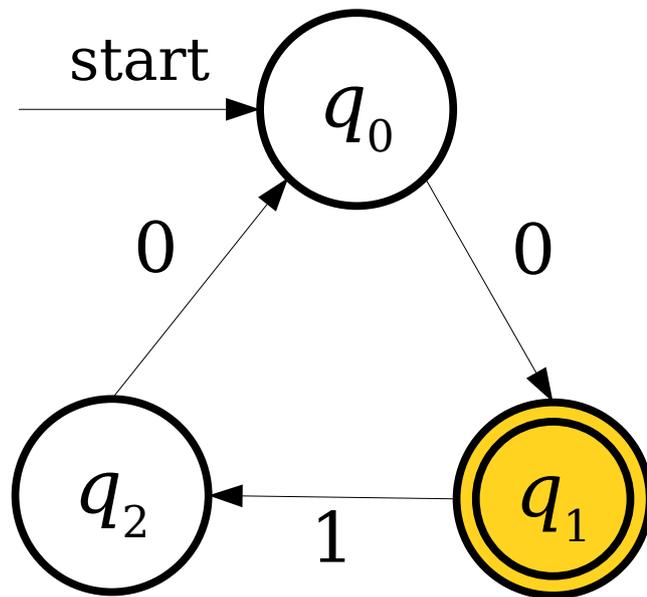
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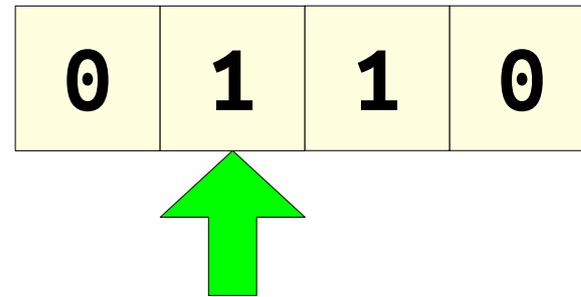
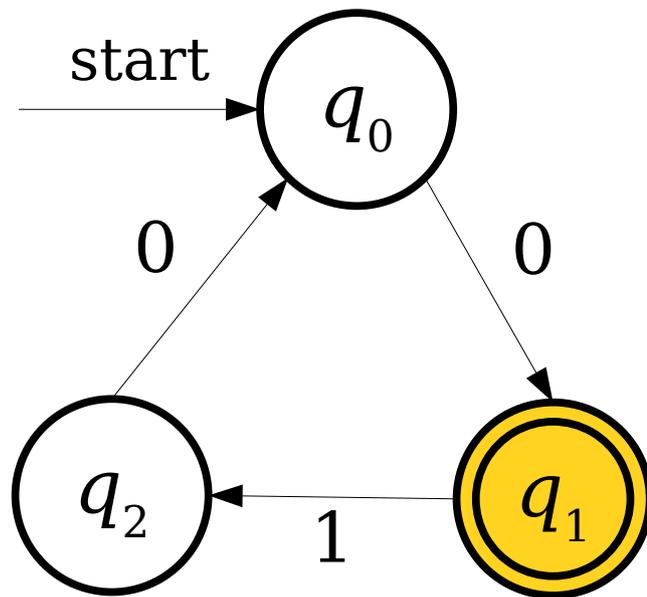
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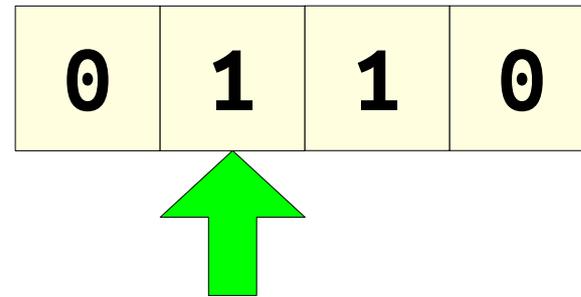
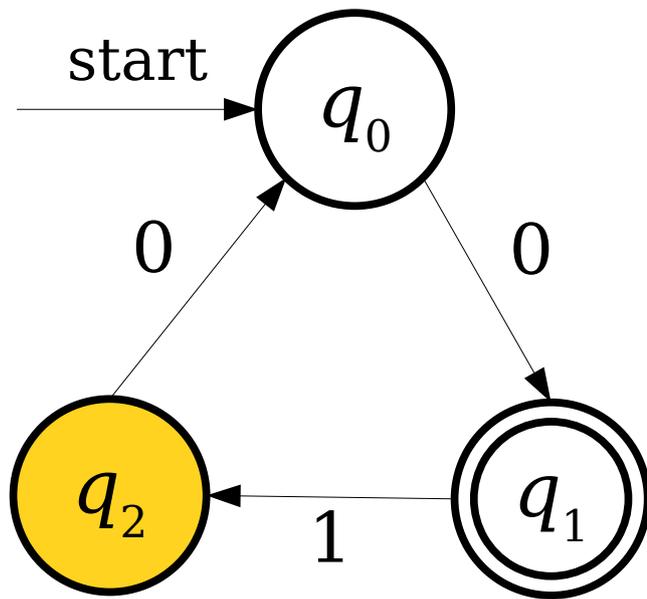
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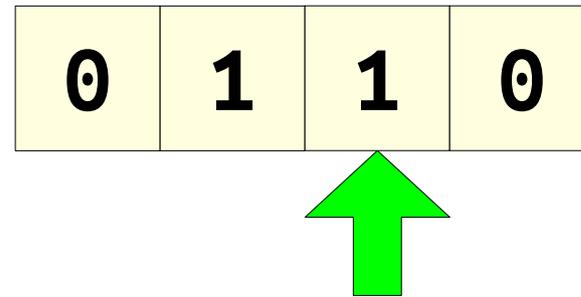
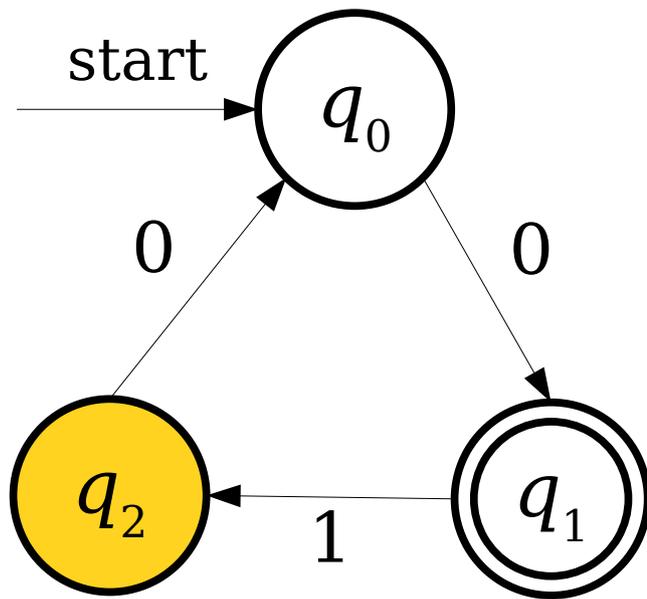
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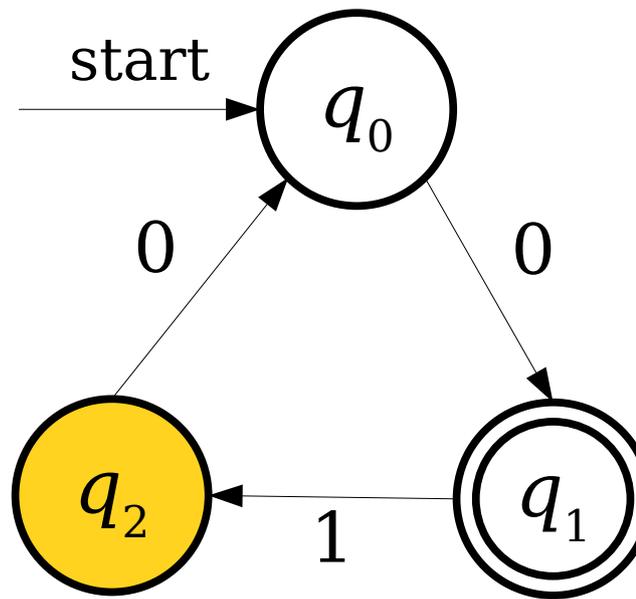
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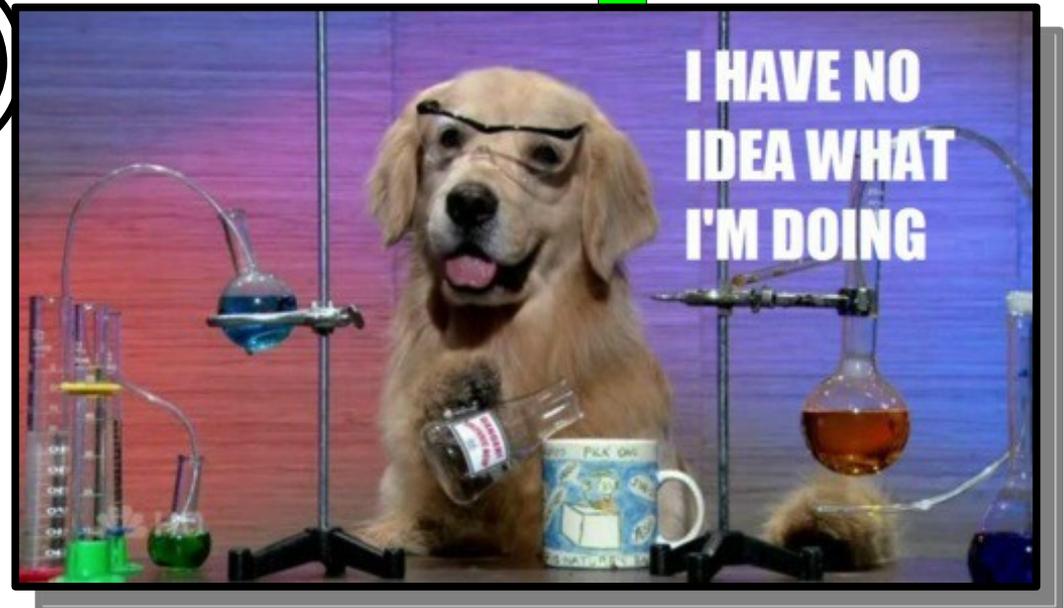
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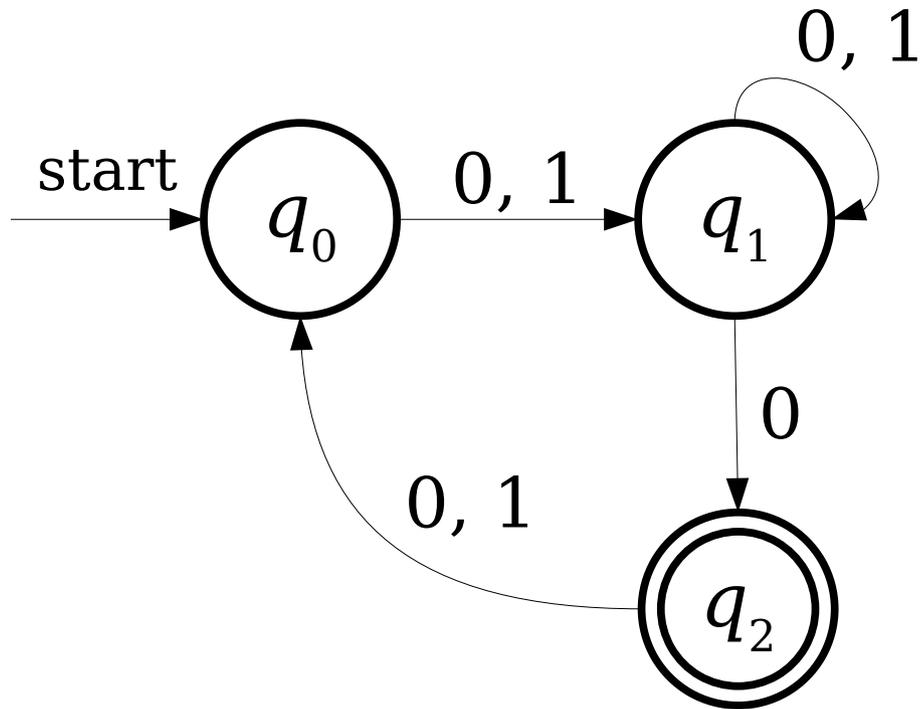
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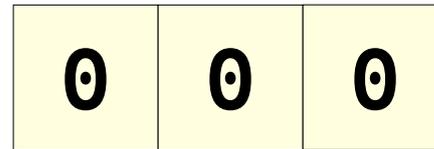
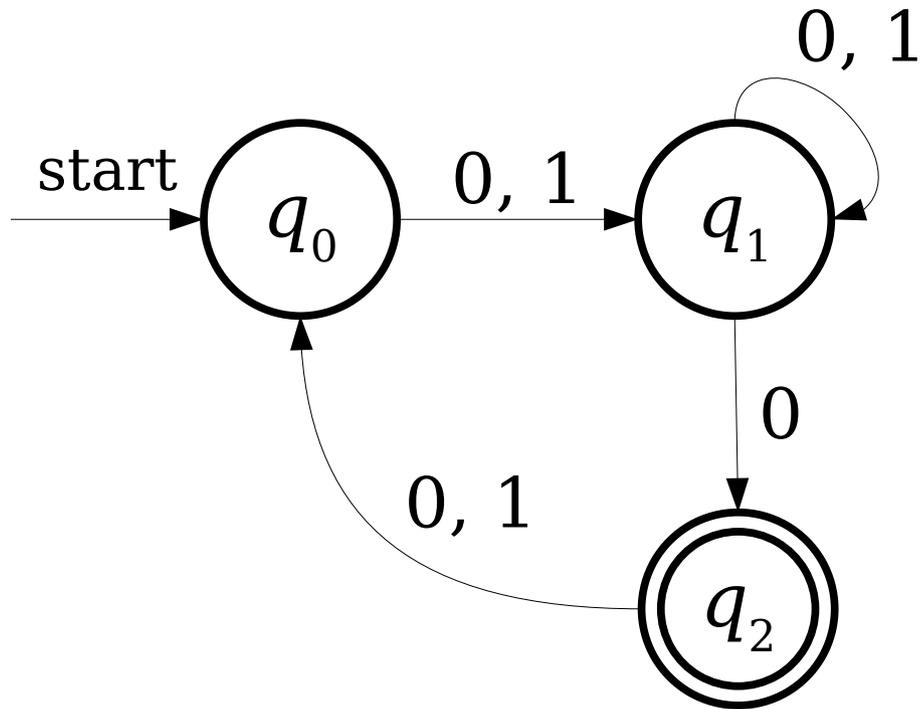
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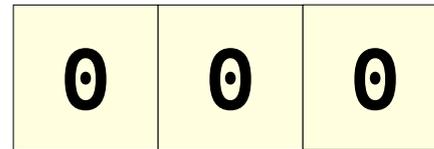
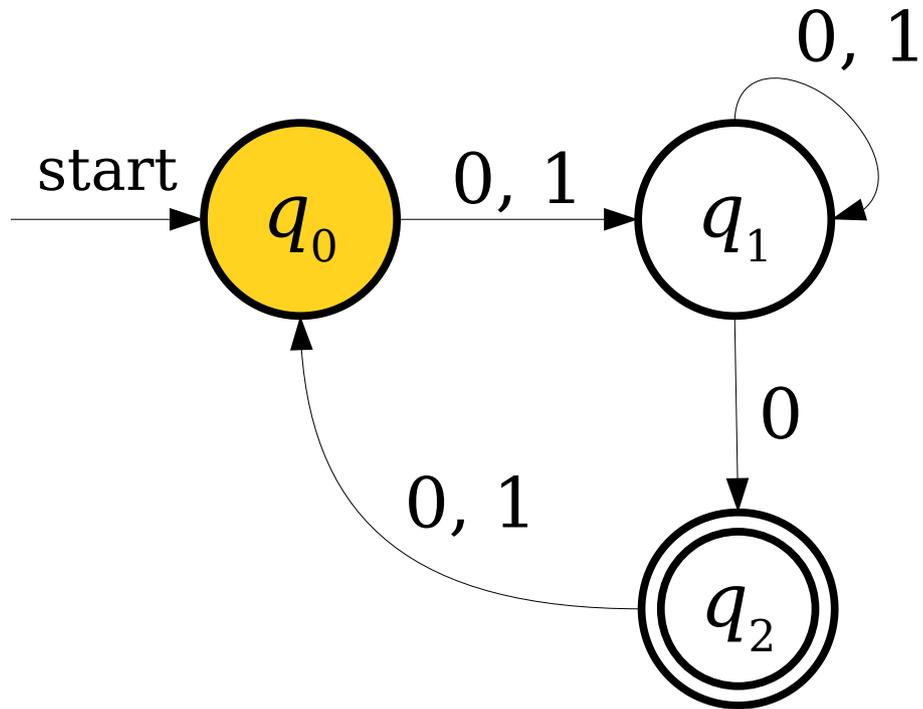
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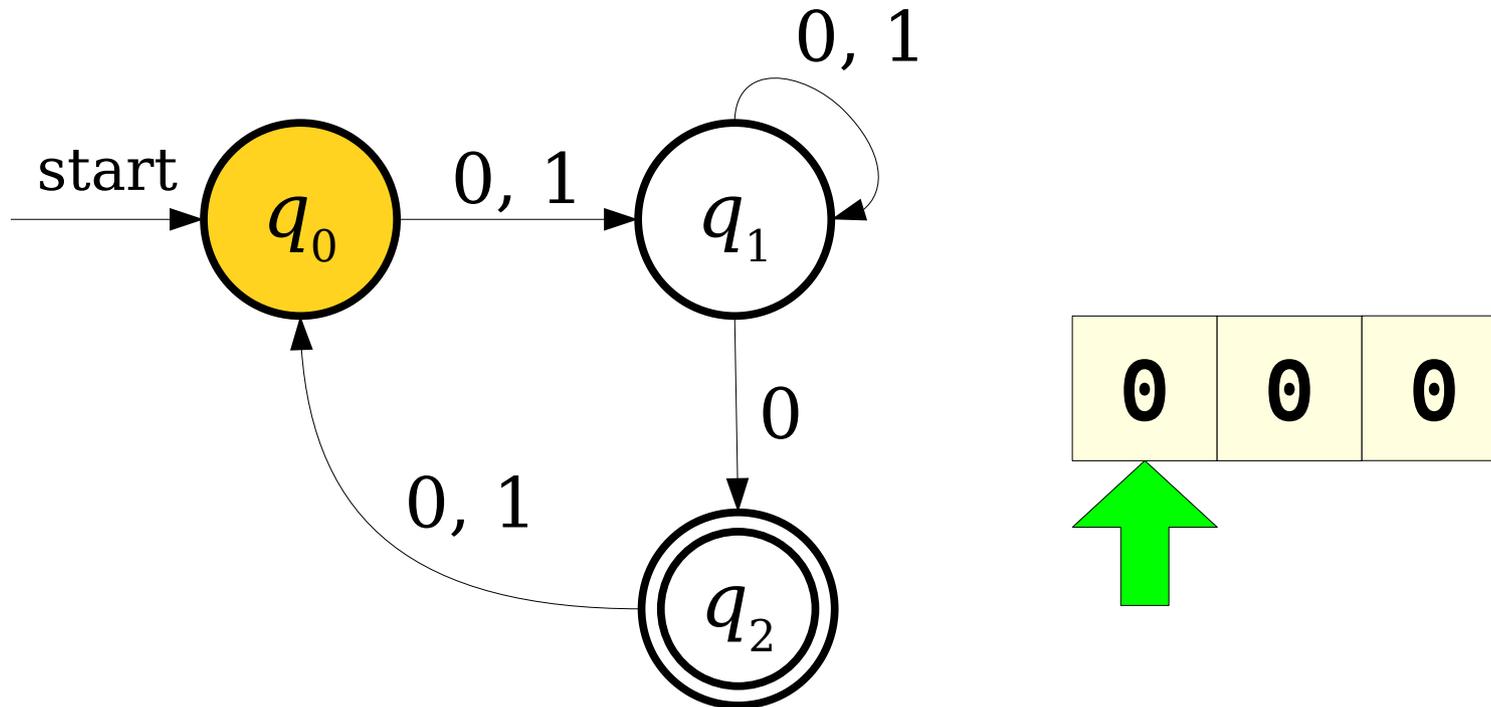
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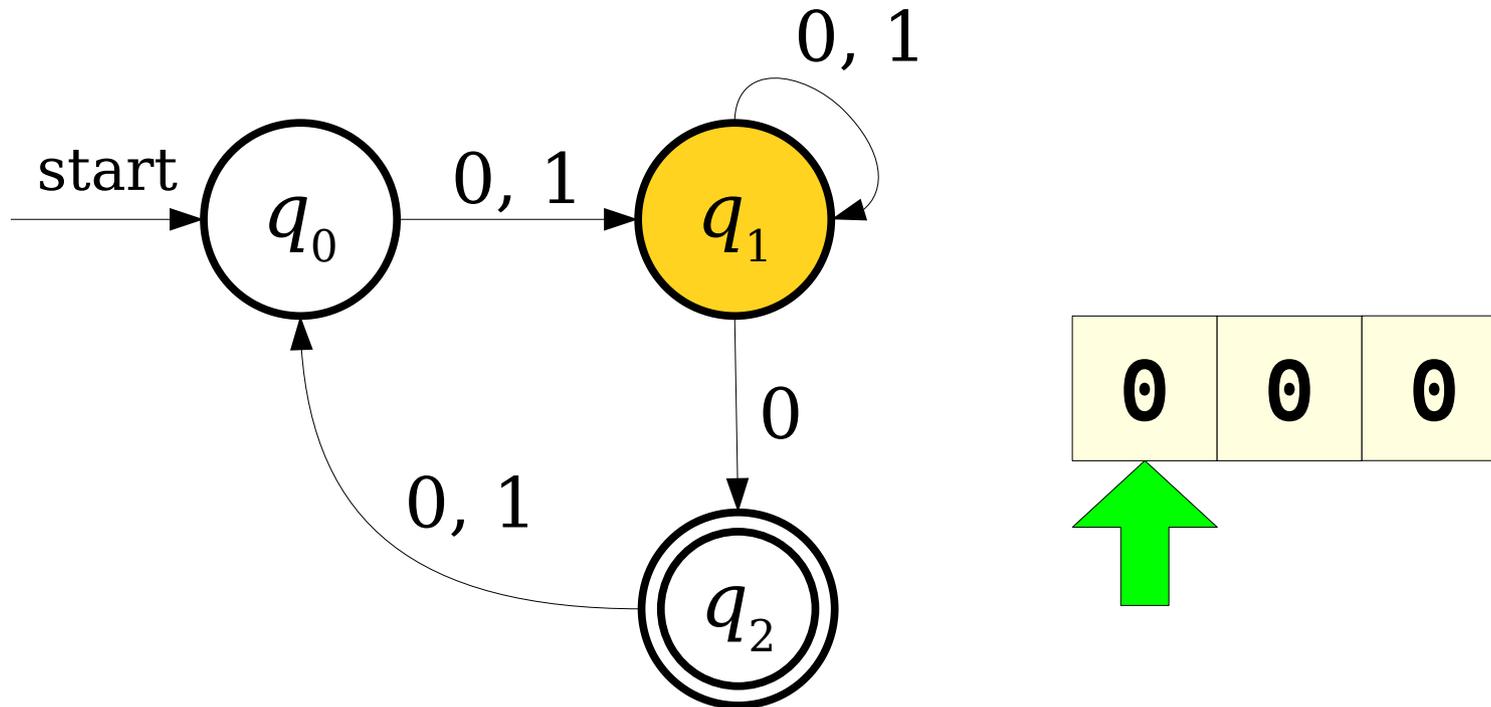
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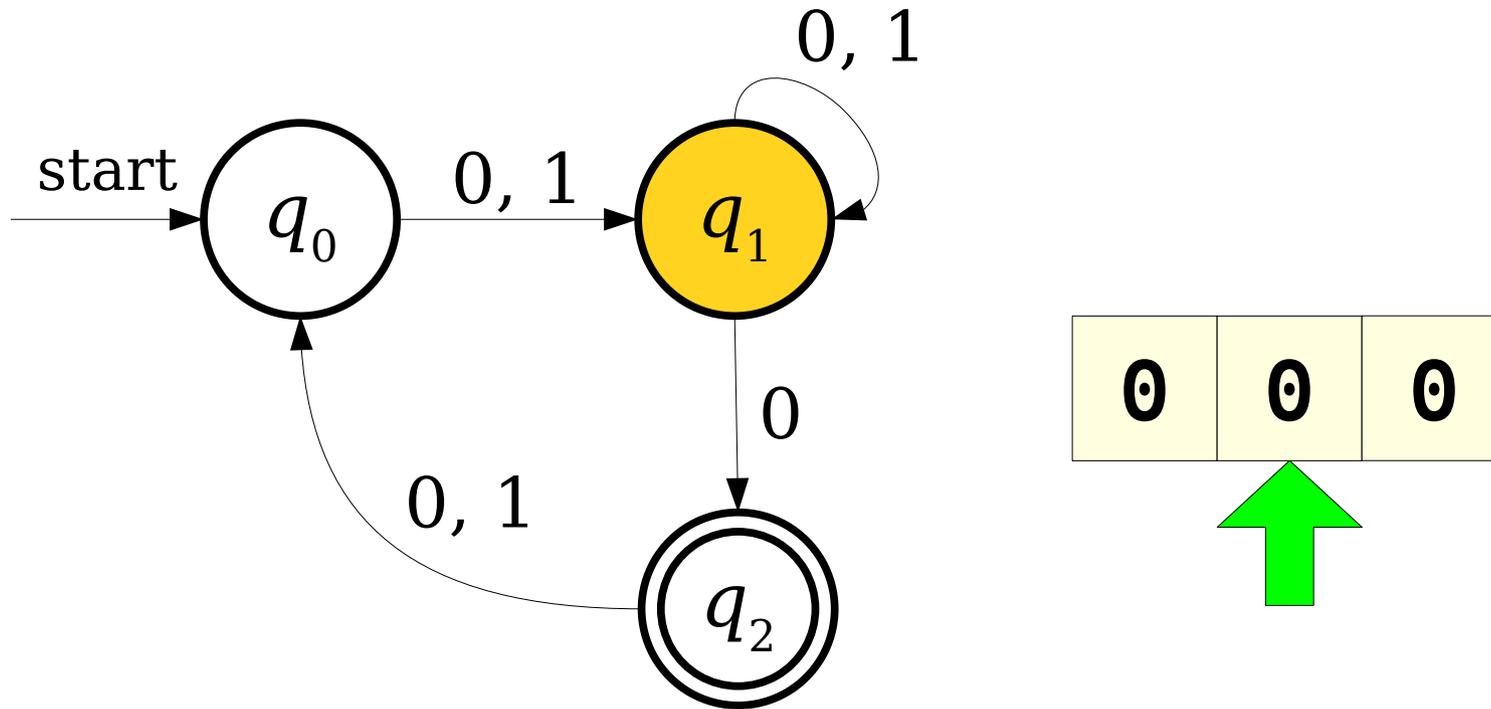
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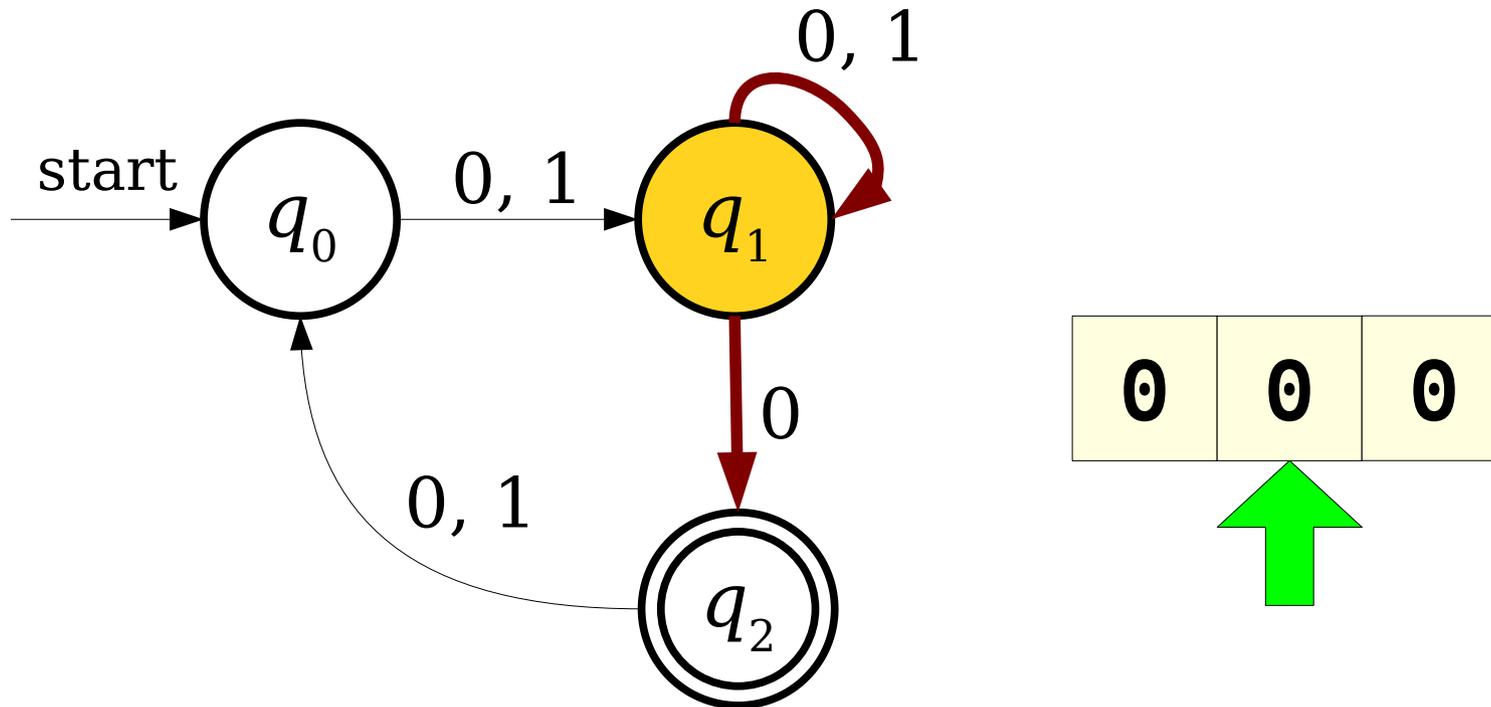
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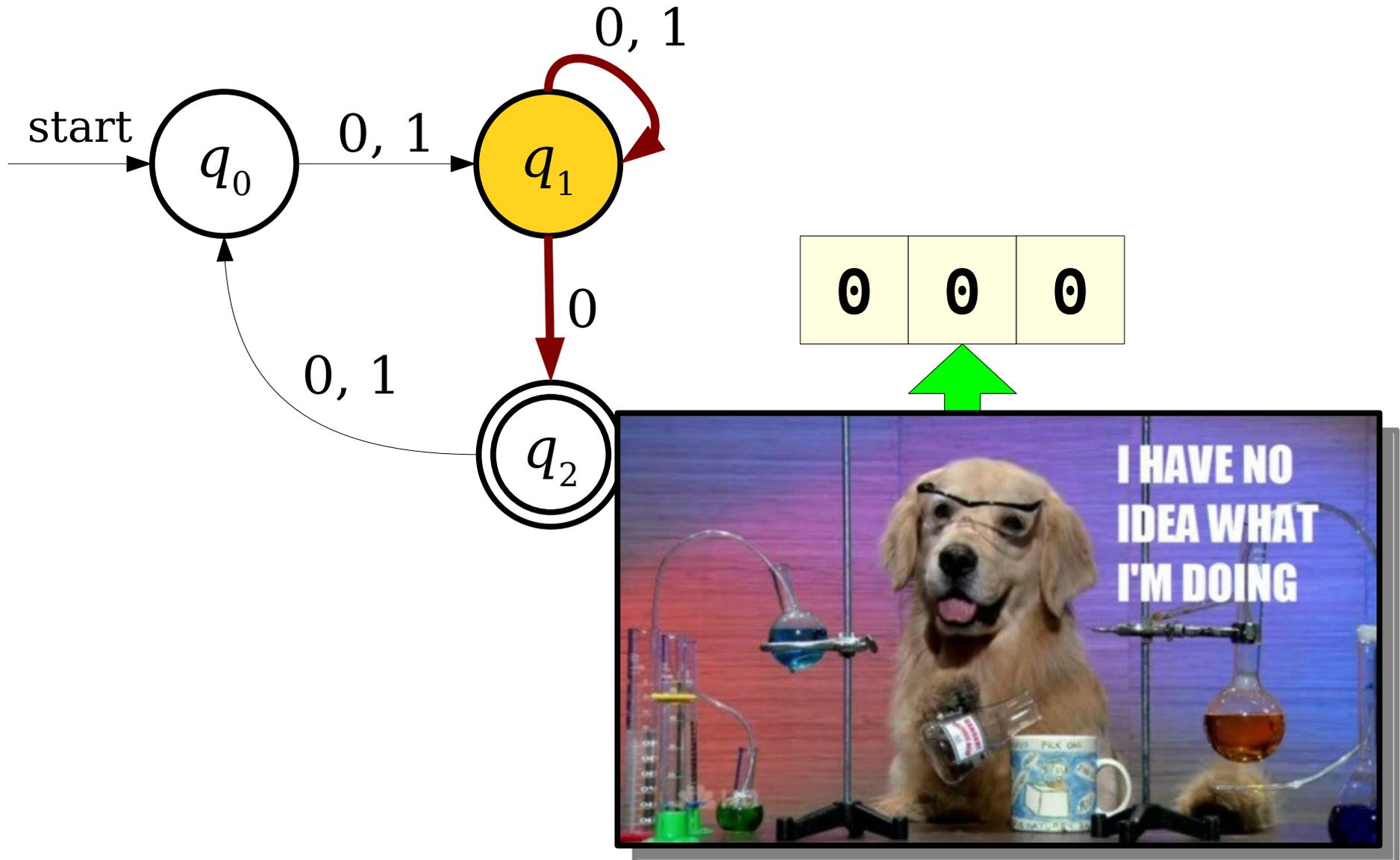
Another Small Problem



Another Small Problem



Another Small Problem



The Need for Formalism

- In order to reason about the limits of what finite automata can and cannot do, we need to formally specify their behavior in *all* cases.
- All of the following need to be defined or disallowed:
 - What happens if there is no transition out of a state on some input?
 - What happens if there are *multiple* transitions out of a state on some input?

Finite Automata

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DFAs

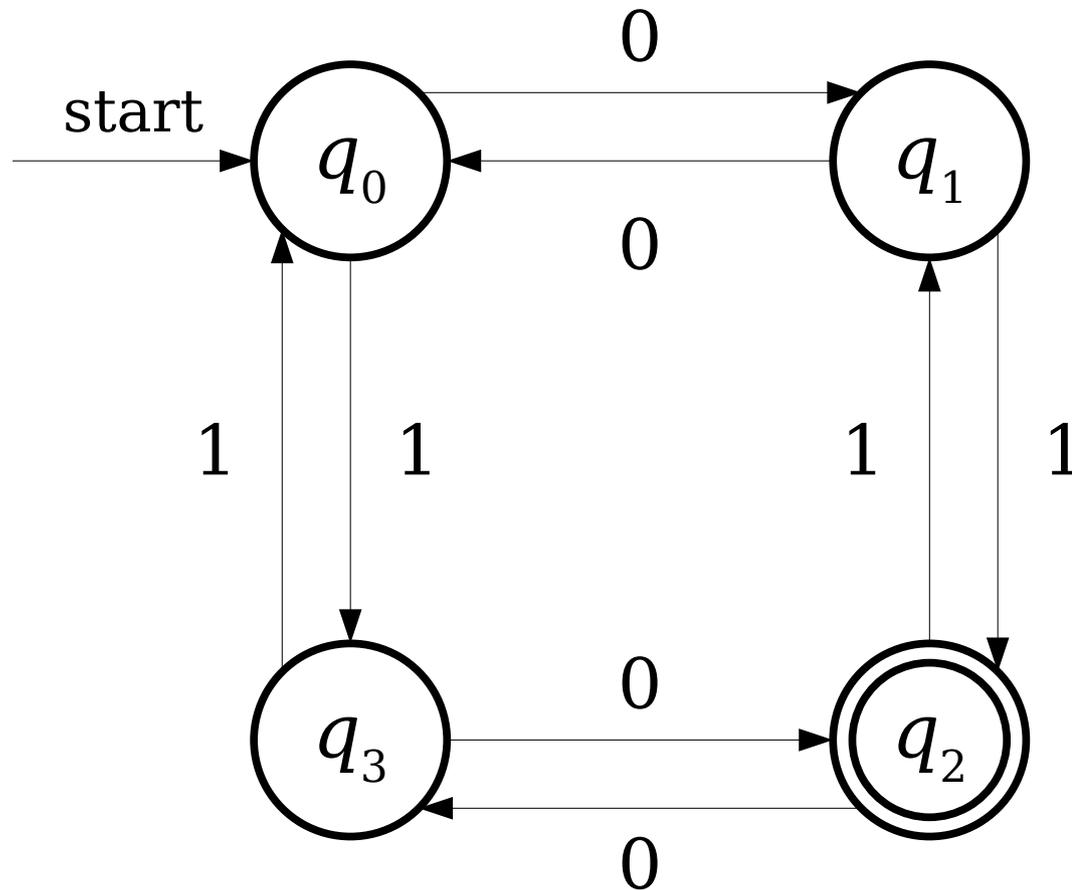
- A ***DFA*** is a
 - ***D***eterministic
 - ***F***inite
 - ***A***utomaton
- DFAs are the simplest type of automaton that we will see in this course.

DFA's

- A DFA is defined relative to some alphabet Σ .
- For each state in the DFA, there must be *exactly one* transition defined for each symbol in Σ .
 - This is the “deterministic” part of DFA.
- There is a unique start state.
- There are zero or more accepting states.

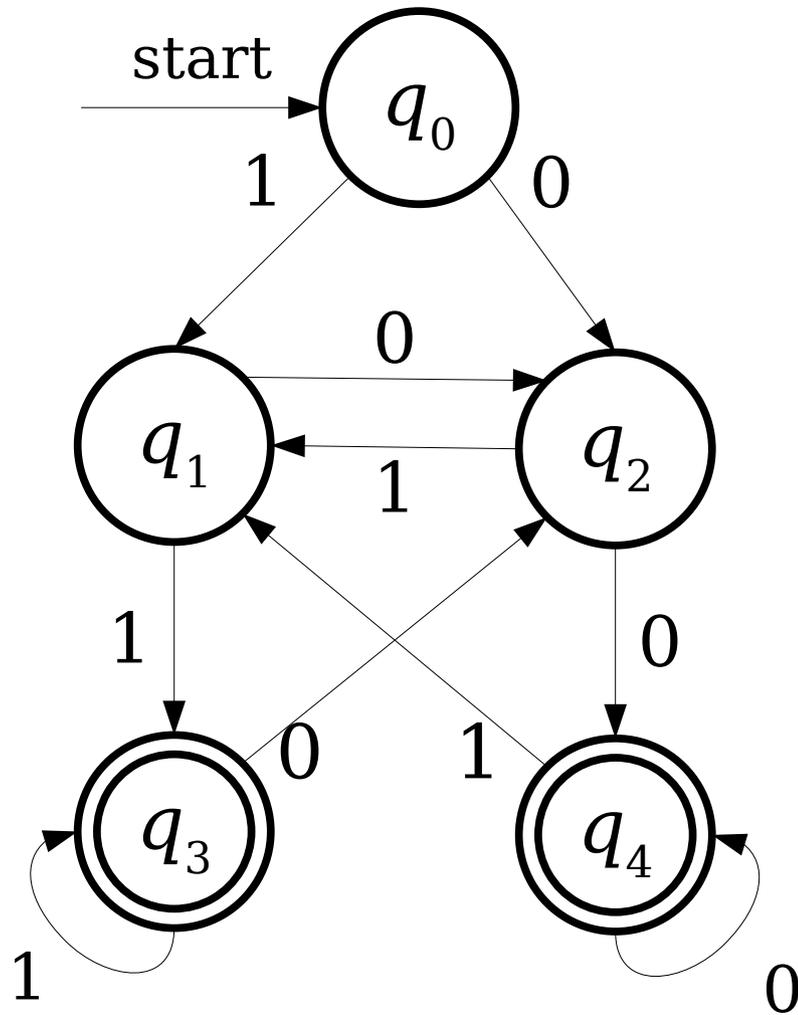
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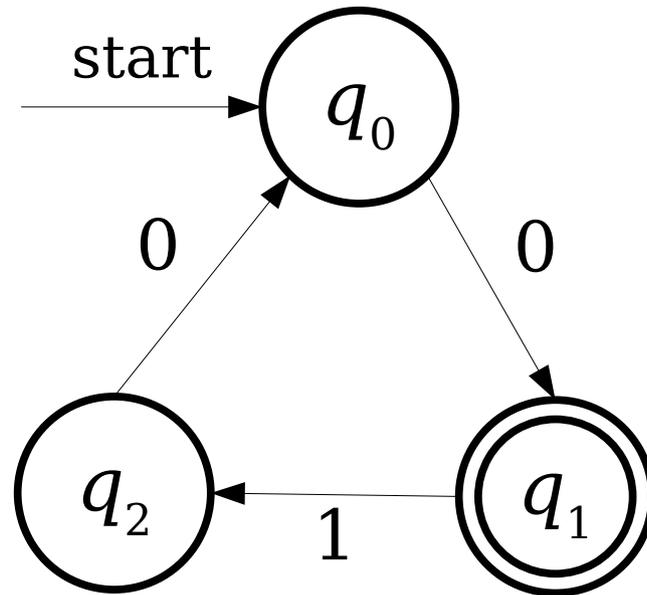
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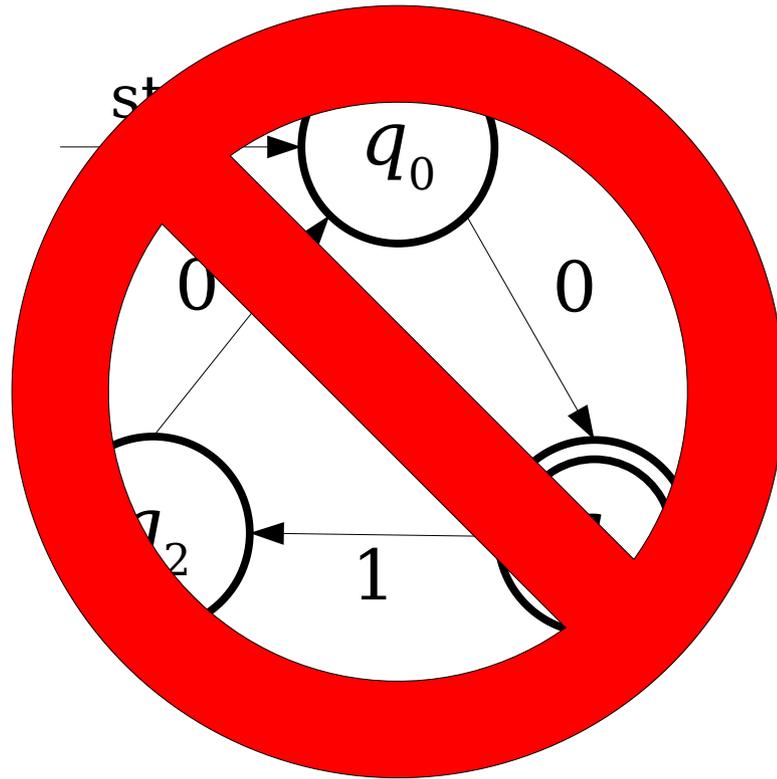


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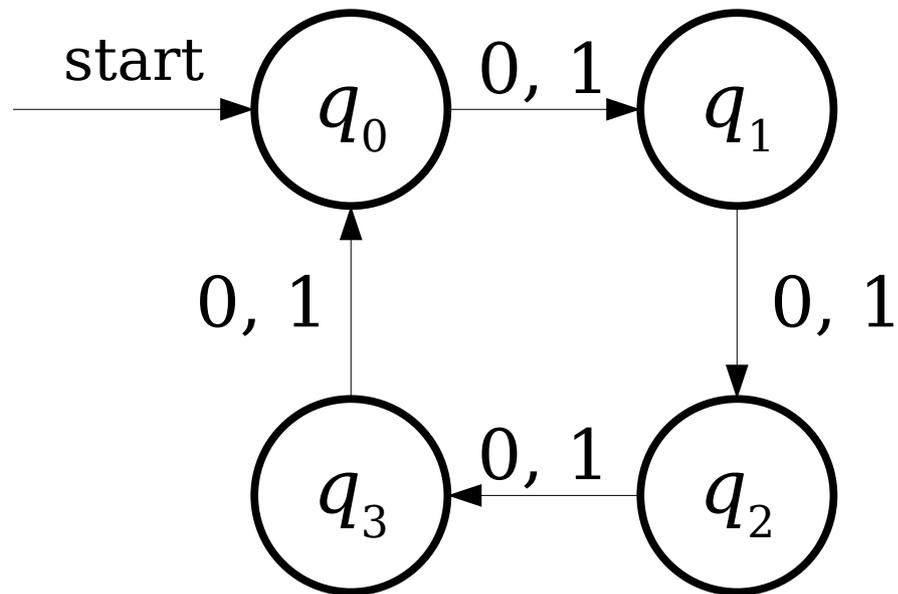


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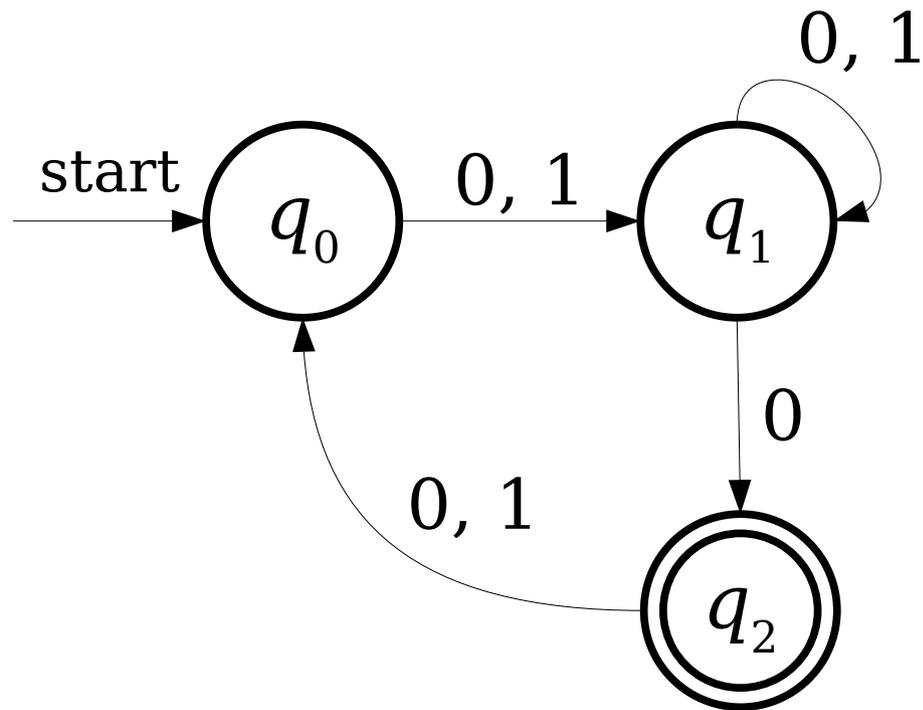
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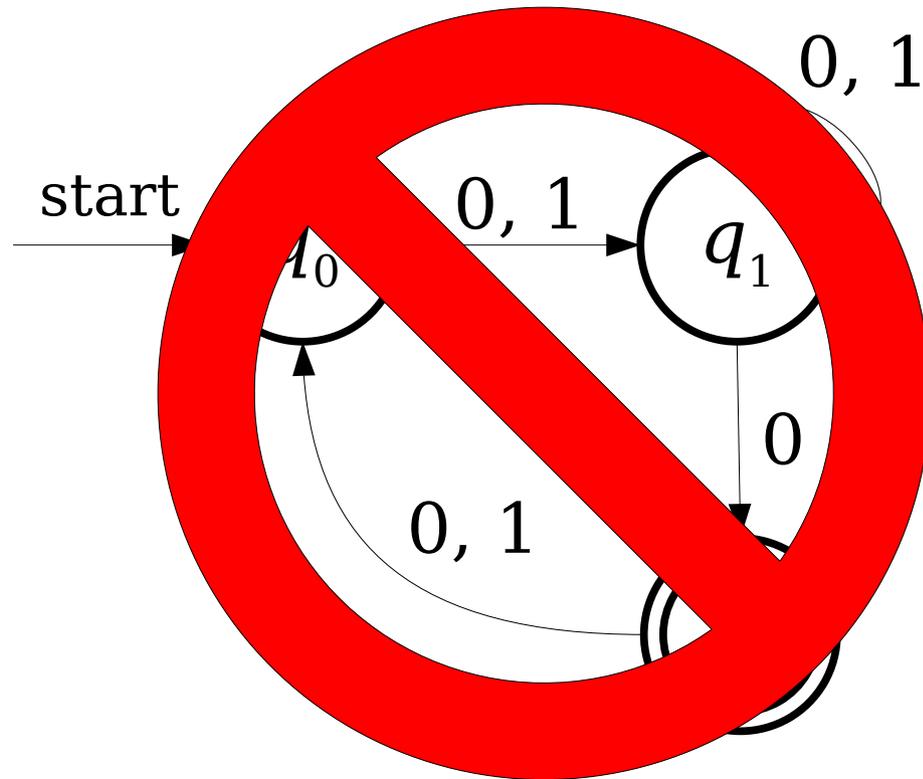


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3. Toward a Model of Computation
4. Automata
5. Languages, Strings, and Alphabets
6. A Small Problem
7. DFAs
8. Designing DFAs
9. What's Next?

Finite Automata

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Designing DFAs

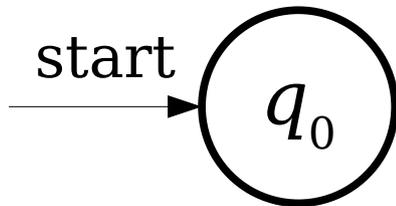
- At each point in its execution, the DFA can only remember what state it is in.
- ***DFA Design Tip:*** Build each state to correspond to some piece of information you need to remember.
 - Each state acts as a “memento” of what you're supposed to do next.
 - Only finitely many different states means only finitely many different things the machine can remember.

Recognizing Languages with DFAs

$L = \{ w \in \{a, b\}^* \mid \text{the number of } b\text{'s in } w \text{ is congruent to two modulo three} \}$

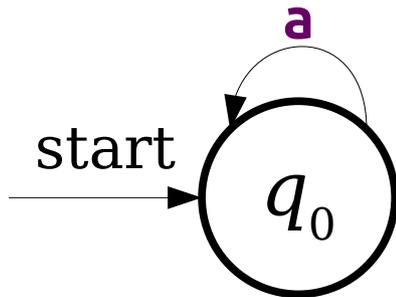
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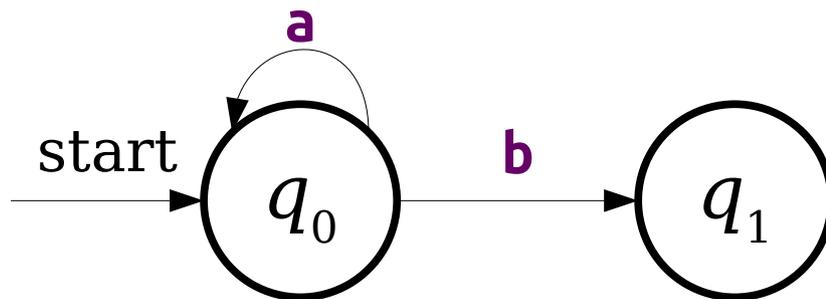
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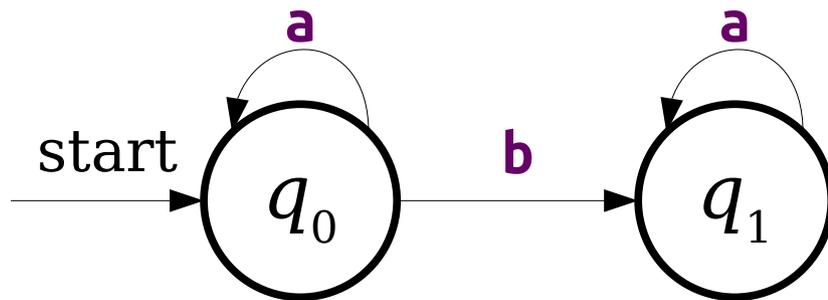
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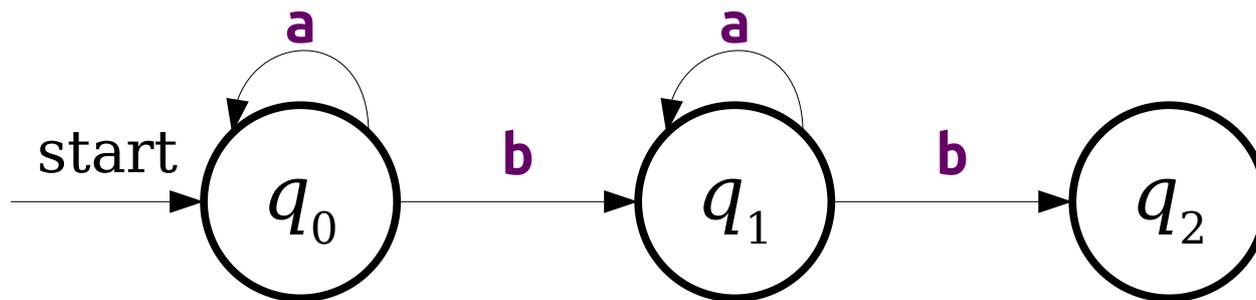
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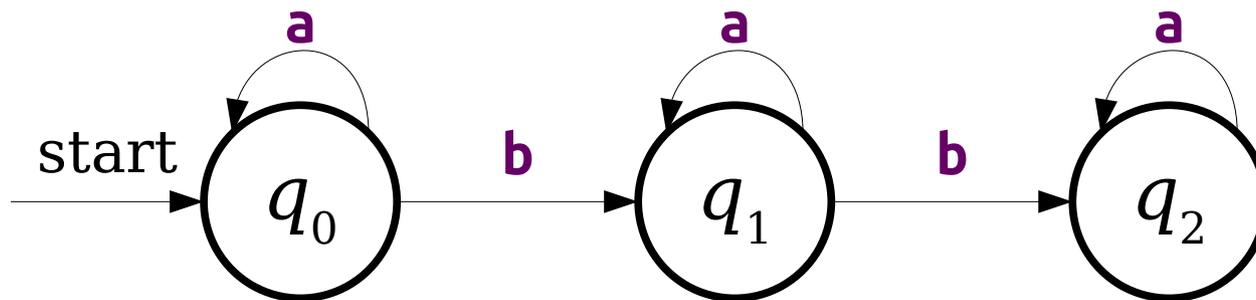
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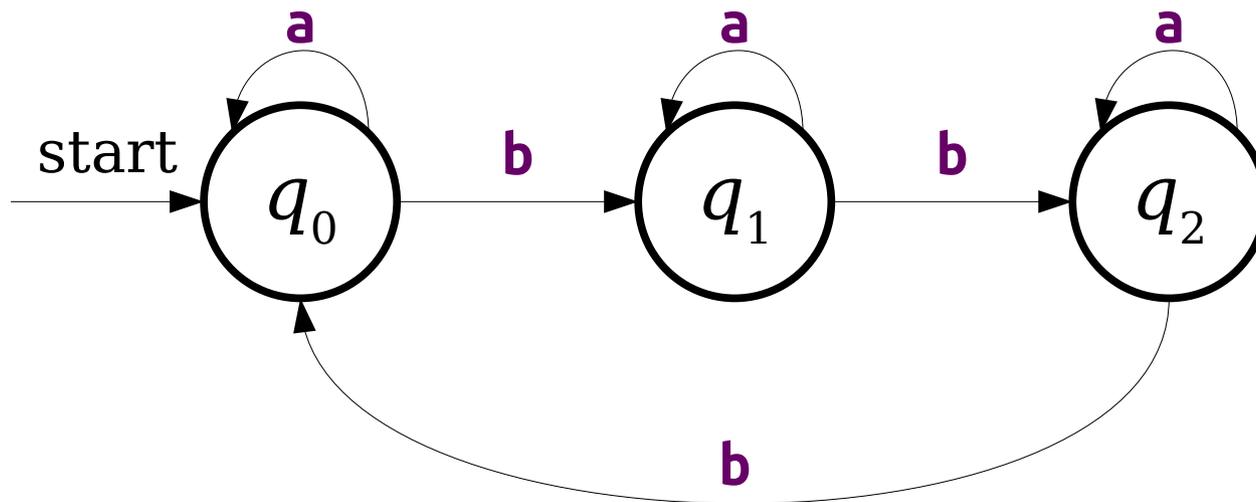
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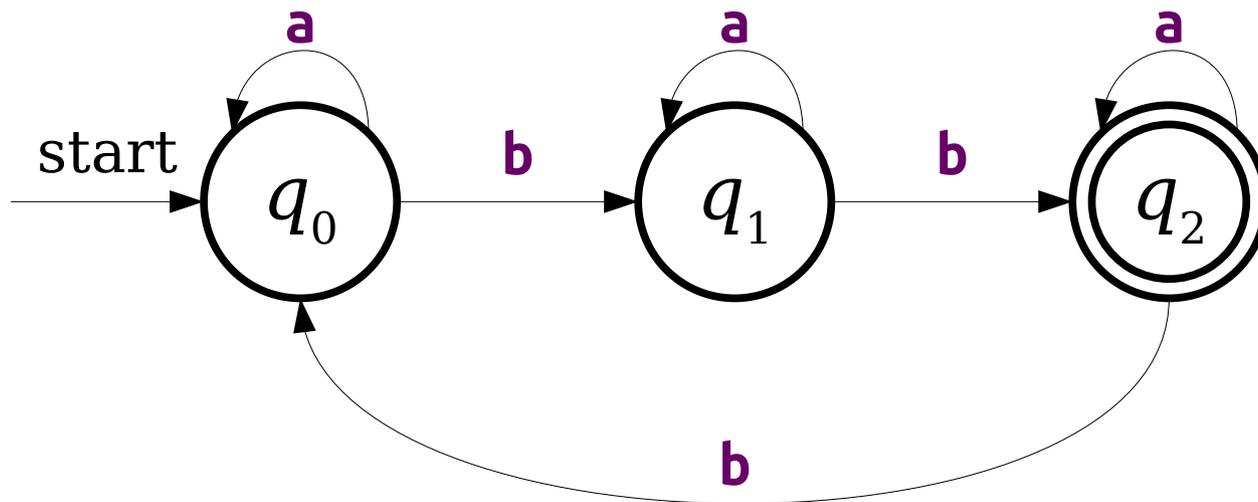
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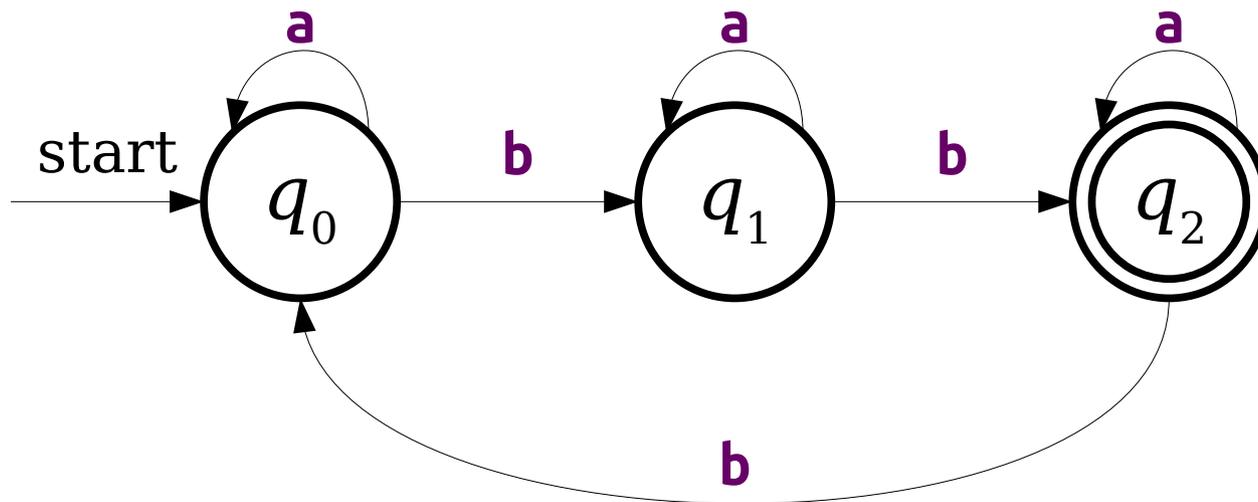
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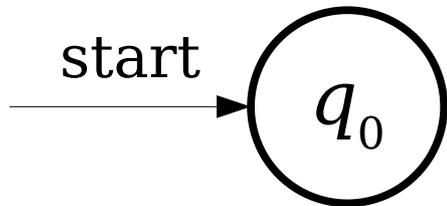
Each state remembers the remainder of the number of **b**s seen so far modulo three.

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$L = \{ w \in \{a, b\}^* \mid w \text{ contains } aa \text{ as a substring} \}$

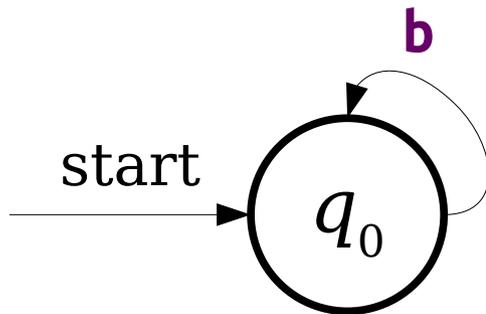
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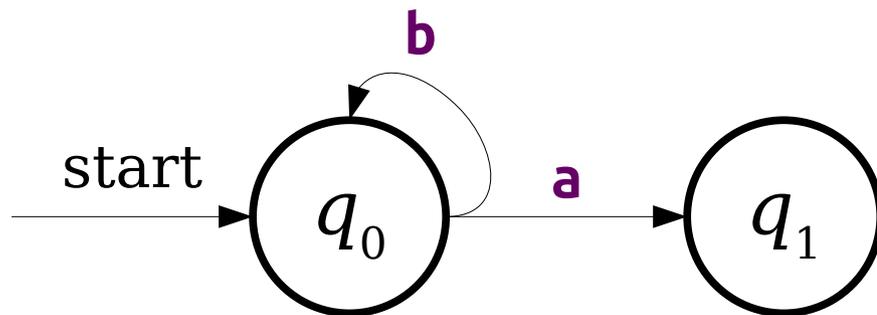
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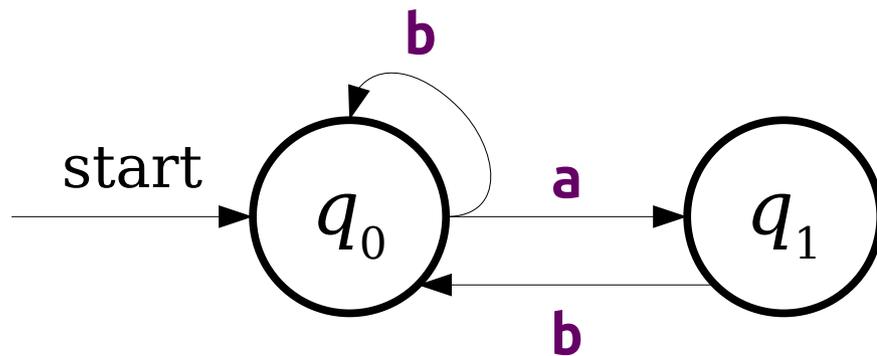
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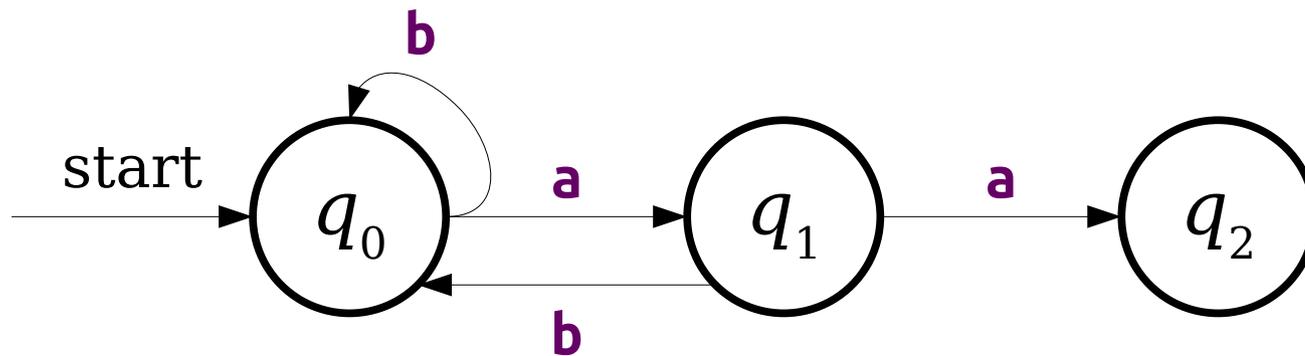
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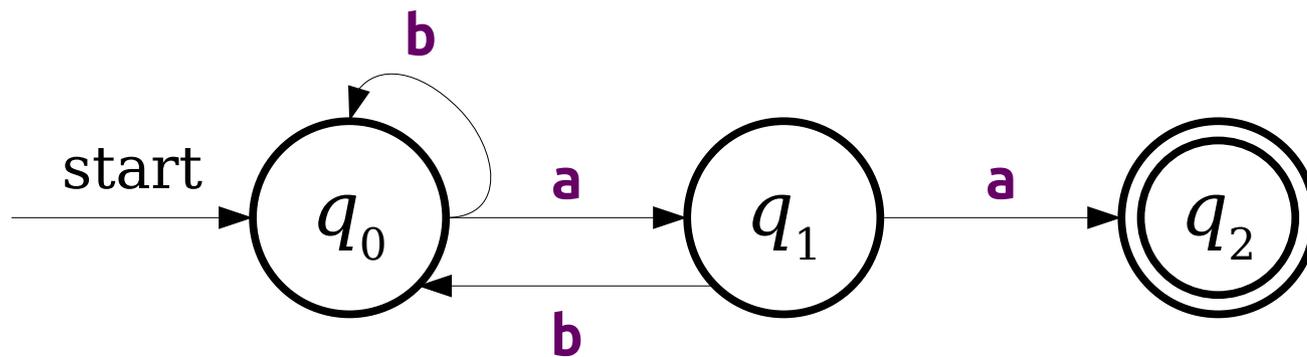
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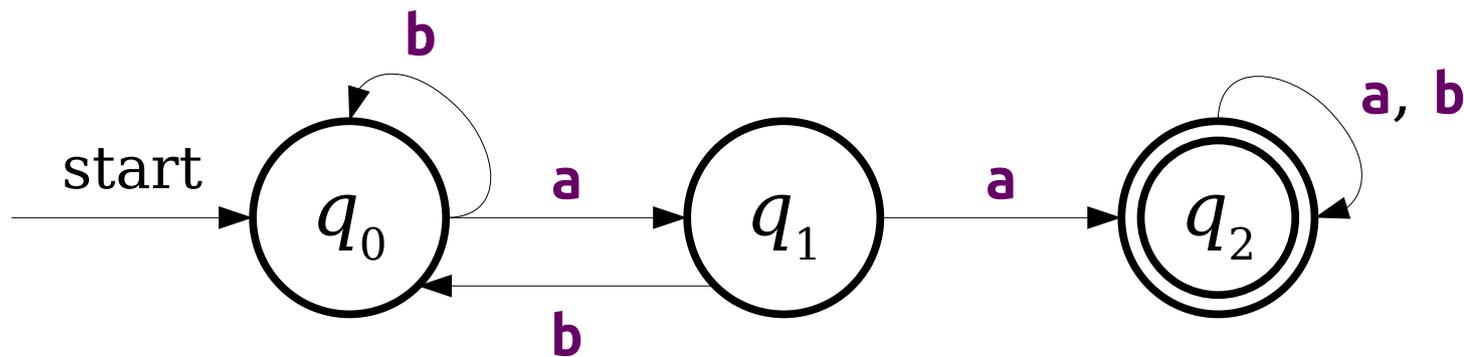
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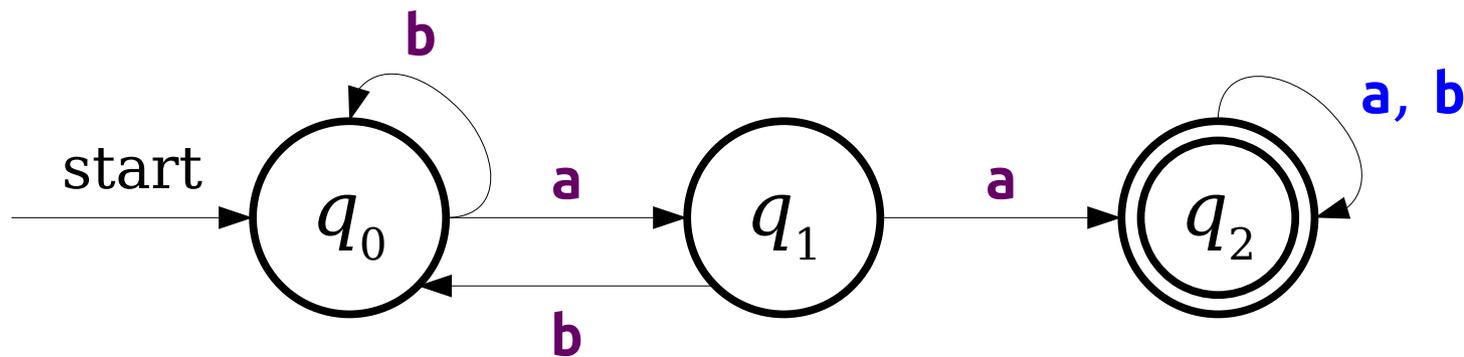
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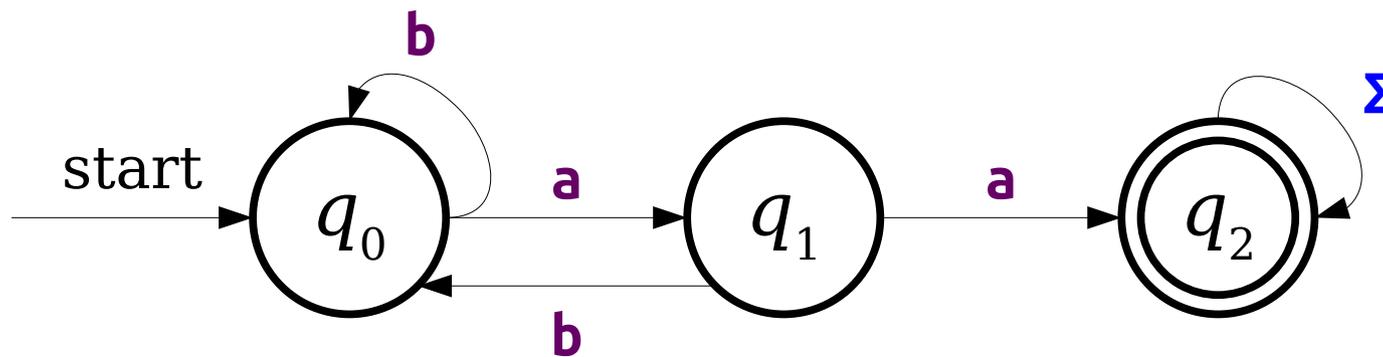
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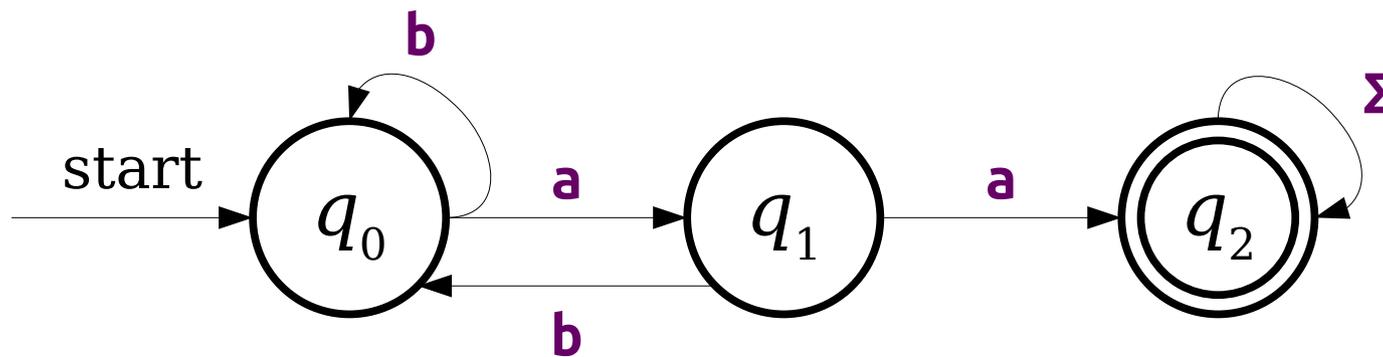
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More Elaborate DFAs

$L = \{ w \in \{a, *, /\}^* \mid w \text{ represents a C-style comment} \}$

Let's have the **a** symbol be a placeholder for "some character that isn't a star or slash."

Try designing a DFA for comments! Here's some test cases to help you check your work:

Accepted:

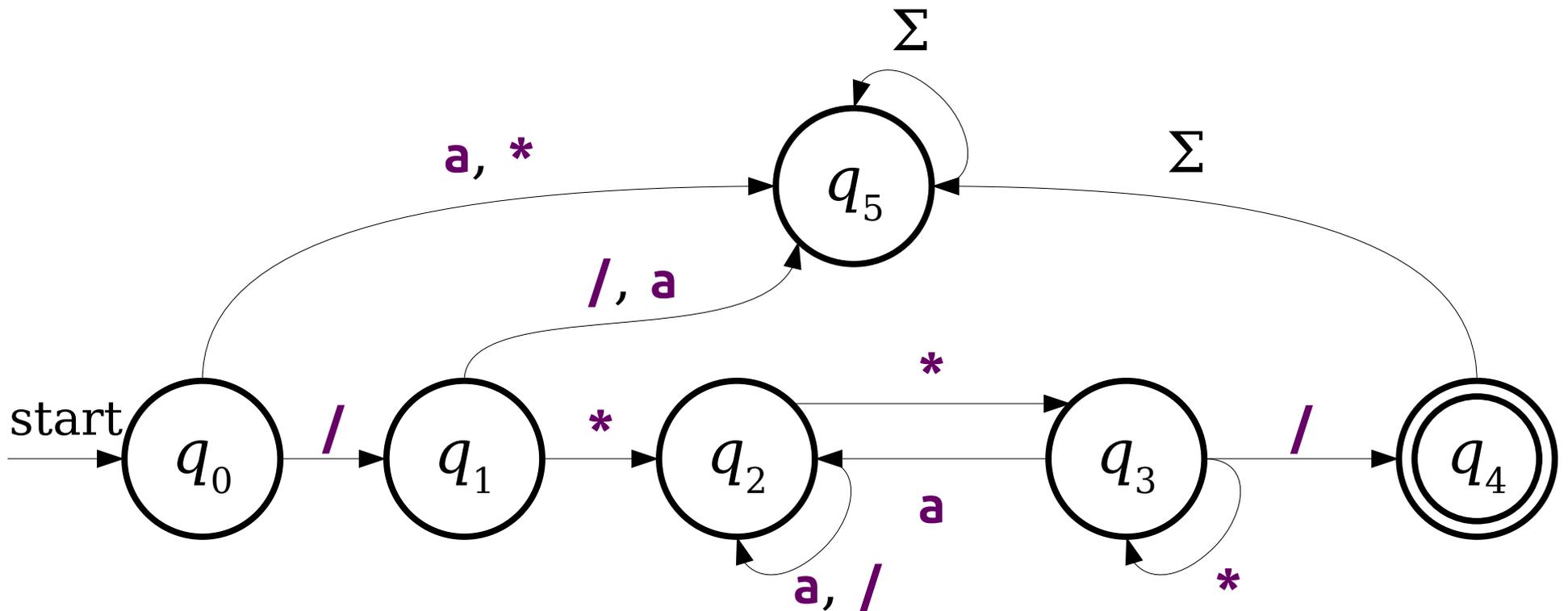
```
/*a*/  
/**/  
/***/  
/*aaa*aaa*/  
/*a/a*/
```

Rejected:

```
/**  
/**/a/*aa*/  
aaa/**/aa  
/*/  
/**a/  
//aaaa
```

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Next Time

- ***Regular Languages***
 - An important class of languages.
- ***Nondeterministic Computation***
 - Why must computation be linear?
- ***NFAs***
 - Automata with Magic Superpowers.